Total number of printed pages: 02 **UG/4th/UMA401**

2023

NUMERICAL METHODS AND CUMPUTER PROGRAMMING

Full Marks: 100

Time: Three hours

The figures in the margin indicate full marks for the questions.

***(Question no: 1** is compulsory and answer **any four** questions.)*

1 a) Given $\frac{dy}{dx} = 1 + x - y^2$, $y(0) = 1$, $h = 0.1$, find the difference in the value of y obtained by Improved Euler Method and Euler Method at $x = 0.1$. 3

b) Given that $\frac{dy}{dx} = 3x + y^2$, with $y(1) = 1.2$ taking $h = 0.1$, find out if there is any improvement in the value of y for $x = 1.1$ obtained by Runge Kutta Fourth Order Method from that of Runge Kutta Third Order. 4

c) Using Successive Approximation Method, find a real root of $x^3 - 5x + 2 = 0$ correct to three decimal places. 4

d) Using Newton-Raphson Method find a real root of $x^3 - e^x + 1 = 0$ correct to four places of decimals. 4

e) Derive Newton-Cote quadrature formula 5

2. a) Using Modified Euler method, find y at $x = 1.2$ and $x = 1.4$ given that $\frac{dy}{dx} = x^3 + \frac{2y}{x}$, $y(1) = 0.5$ with correct result upto four places of decimals. 4+4

b) Using Runge-Kutta method of Fourth order, $\frac{dy}{dx} = \frac{1}{2}(1 + x)y^2$ with $y(0) = 1$ at $x = 0.2, 0.4, 0.6$. 12

3. a) Using Euler's method, find a solution of $\frac{dy}{dx} = x + \frac{y}{x}$, with initial condition $y = 2$ at $x = 1$ for the range $1 \leq x \leq 1.8$ in steps of 0.2. Compare the results with analytical solution. 8+4

b) Use Milne's method to compute $y(0.8)$ and $y(1.0)$ if $\frac{dy}{dx} = 1 + y^2$ with $y(0) = 0$, $y(0.2) = 0.2027$, $y(0.4) = 0.4228$, $y(0.6) = 0.6841$, with correct result upto four places of decimals. 4+4

4. a) Under what situation Newton's forward derivative formula is applied? 2+8

using the suitable derivative formula find the first and second order derivative of f(x) at x=1.5 if

X:	1.5	2.0	2.5	3.0	3.5	4.00
f(x):	3.375	7.00	13.625	24.000	38.875	59.000

b) Use Simpson's 1/3 th rule to evaluate $\int_0^1 \frac{dx}{x^3+x+1}$ 5

c) Using Trapezoidal rule estimate the integral $\int_0^2 e^{x^2} dx$ taking 12 intervals 5

5. a) Under what situation Newton's backward derivative formula is applied? Using appropriate formula find the value of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at x=1.25 if 2+8

X:	1.00	1.05	1.10	1.15	1.20	1.25	1.30
Y:	1.000	1.025	1.049	1.072	1.095	1.118	1.140

b) Evaluate $\int_4^{5.2} log x \, dx$ by (i) Trapezoidal and (ii) Simpson's 3/8 th rule. Also compare the results with its integral actual. 4+4+2

6. a) Using Bisection Method find a real root of $x^3 - 3x + 1 = 0$ correct up to three decimal places. 7

b) Using Regula-Falsi Method find a real root of $x^3 - 4x - 9 = 0$ correct to three decimal places. 7

c) Using Secant Method find a real root of $x^3 - 4x + 2 = 0$ correct up to three decimal places. 6

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