Total number of printed pages: **2** **UG/3rd /UMA302/July-Dec 2024**

2024

DISCRETE MATHEMATICS

Full Marks: 100

Time: Three hours

The figures in the margin indicate full marks for the questions.

*Answer **any five** questions.*

1. a) Find the number of lines of a complete graph K_p. 5

   b) Show that self-complementary graph has 4n or 4n + 1 vertices. 5

   c) If G is a graph in which the degree of every vertex is at least two then G contains a cycle. 5

   d) Prove that the finite non-empty poset $(A, \leq)$ has at least one maximal and one minimal element in A. 5

2. a) Show that every cubic graph has an even number of points. 4

   b) The edge connectivity of a graph G cannot exceed the minimum degree of a vertex in G. i. e. $\lambda(G) \leq \delta(G)$ 6

   c) Consider a lattice L. Prove that the relation $a \leq b$ defined by either $a \wedge b = a$ or $a \vee b = b$ is a partial ordering on lattice L. 5

   d) Show that in a Boolean algebra, the complement of every element is unique. 5

3 a) Prove that every distributive lattice is modular. 5

   b) For any Boolean algebra $(B, \vee, \wedge, ', 0, 1)$ prove that $(a \vee b)' = a' \wedge b'$ and $(a \wedge b)' = a' \vee b'$, for all a, b in B. 5

   c) Find the CNF of the Boolean expression by truth table method $f(x, y, z) = (x \wedge y')$ 5

   d) Find the DNF of the Boolean expression by truth table method $f(x, y, z) = (x \wedge y) \vee (y \wedge z')$ 5

4. a) Prove that for a bounded distributive lattice L, the complement are unique if they exist. 2X2 = 4

   b) Give an example to show that a modular lattice may not be distributive. 4

   c) A connected graph G has an Eulerian trail iff it has at most two odd vertices. That is, it has either no vertices of odd degree or exactly two vertices of odd degree. 6

   d) Define bipartite graph. Prove that a graph which contains a triangle cannot be bipartite. 1+5=6

5. a) Show that a graph is a tree if and only if every two points in it is joined by a unique path. 3+3=6

   b) Prove that a tree T with n vertices has (n – 1) edges. 4

c) Prove that the power set of S under $\subseteq$, is a poset. Draw the Hasse diagram of the poset $(P(S), \subseteq)$, where $S = \{a, b, c, d\}$. 4+2=6

d) Define isomorphism between lattices. Show that the lattice $L = \{1, 2, 3, 6\}$ under the divisibility and the lattice $(P(S), \subseteq)$ where $S = \{a, b\}$, are isomorphic. 4

6. a) Show that the degree of a vertex of a simple graph on n-vertices cannot exceed $(n - 1)$. Does there exists a 4-regular graph on 6 –vertices? If so draw such graph. 3+2 =5

b) Define acyclic graph. If $G(p, q)$ is a connected graph in which $p = q + 1$, show that G is acyclic. 1+4 = 5

c) Define totally ordered set. 1+ 4= 5

Give one example of each of the following:

(i) Symmetric but not anti-symmetric

(ii) anti-symmetric but not symmetric

(iii) Both symmetric and anti-symmetric

(iv) Neither symmetric nor antisymmetric

d) Give an example (with justification) of a poset A and a non-empty subset S of A such that S has lower bounds in A but glb(S) does not exist. 5

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