Total number of printed pages: 02 **UG /3rd Semester/UMA302**

2023

DISCRETE MATHEMATICS

Full Marks: 100

Time: Three hours

The figures in the margin indicate full marks for the questions.

*Answer **any five** questions.*

1. a) Prove that $A-(B\cup C)=(A-B)\cap(A-C)$ 5

   b) If $f: A \to B$ and $g: B \to C$ be two invertible functions then show that $g_o f: A \to C$ is also invertible. 5

   c) If I be the set of all integers, then show that the relation R on I defined by ${}_xR_y$ if and only if x-y is divisible by 3, (for all integers x and y) is an equivalence relation. 5

   d) Show that the set $G=\{1, -1, i, -i\}$ is a group under multiplication, where i is the imaginary unit. 5

2. a) Define coset in a group. Give an example with justification. 3

   b) Prove that the inverse of the product of two elements of a group is the product of their inverses in reverse order. 5

   c) Show that intersection of subgroups of a group is a subgroup. Provide an example to illustrate that the union of two subgroups may not necessarily be a subgroup. 5 + 2

   d) Show that if (B, +, . ,/) is a Boolean Algebra, then show that (i) $x+x=x$ and (ii) $(x^/)^/=x$, for every $x\in B$. 5

3. a) Find the Disjunctive Normal Form (DNF) of the Boolean function $f(x,y,z)=x+yz$. 6

   b) Show that the set R of all real numbers is an integral domain. 7

   c) Define and provide one example for each of the following 7

      (i) Normal Subgroup (ii) Homomorphism of a group (iii) Field

4. a) Write the negation of each of the following statements 1+1+1=3

      (i) He swims if and only if the water is warm.

(ii) If he studies, he will pass the examination.

(iii) 2 +4 = 6 and 7 < 11

b) Examine whether the following proposition is tautology: 5

$p \vee [\sim p \rightarrow (q \vee (q \rightarrow (\sim r))]$

c) Define minterm and maxterm of two statement variables. Find the Principal Disjunctive Normal Form of the following compound proposition: 2+5= 7

$(p \wedge \sim q \wedge \sim r) \vee (q \wedge r)$

d) Represent the following argument: 2+3=5

If the last digit of this number is a 5, then this number is divisible by 5.

The last digit of this number is a 5.

....

This number is divisible by 5.

Symbolically and determine whether the argument is valid.

5. a) Construct the truth table of the following compound proposition 4

$(p \rightarrow q) \vee (p \rightarrow r) \leftrightarrow (q \wedge r)$

b) Is there a simple graph corresponding to the following degree sequences? 2+2=4

(i) 1, 1, 2, 3

(ii) 2, 2, 4, 6

c) Show that self-complementary graphs have 4n or 4n -1 vertices 5

d) Define kernel of a homomorphism between groups. 2 +5 =7

Let G and H be two groups with identities e and e', respectively.

If f: G → H is a homomorphism, the prove that the kernel of f is an normal subgroup.

6 a) Show that the maximum number of edges in a simple graph with m vertices is m(m-1)/2. 5

b) Prove that the edge connectivity of a graph G cannot exceed the minimum degree of a vertex in G. 5

c) Define Eulerian graph. Give an example of a graph which is Hamiltonian but not Eulerian and vice-versa. 1+2+2 = 5

d) Show that every cubic graph has an even number of vertices. 5

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