Total number of printed pages: **3** **UG/3rd/UMA302**

2022

DISCRETE MATHEMATICS

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

Answer ***any five*** *questions.*

1. a) State *true* or *false* : 1x 5=5

i) The ring of all integers is an Integral domain.

ii) If S be the set of people, then the relation R on S defined by $_xR_y$ if x is father of y is Symmetric.

iii) The function $f: R \to R$ defined by $f(x) = x^2$ is many-one into.

iv) In the multiplicative group $\{1, w, w^2\}$, the order of w is 2.

v) The inverse of every element in a group is unique.

b) Fill up the blanks: 1x 5=5

i) If the function $f: A \to B$ is invertible, then its inverse is……………

ii) Every subgroup of an abelian group is ………………..

iii) The order of each subgroup of a finite group is a ……… of the order of the group.

iv) Every cyclic group is ………………..

v) Every field is an …………………..

c) Prove that $A - (B \cup C) = (A - B) \cap (A - C)$ 5

d) If $f: A \to B$ and $g: B \to C$ be two invertible functions then show that $gof: A \to C$ is also invertible. 5

2. a) Show that the set $G = \{1, -1, i, -i\}$ is a group under multiplication. 5

b) If I be the set of all integers, then show that the relation R on I defined by $_xR_y$ if and only if x-y is divisible by 5, (for all integers x and y) is an equivalence relation. 5

c) Show that, a non-empty subset H of a group G is a subgroup of G if and only if $a \in H, b \in H$ implies $ab^{-1} \in H$. — 8

d) Define cyclic group. Give an example. — 2

3. a) Show that if H and K are two normal subgroups of a group G, then HK is normal subgroup of G if $HK = KH$. — 5

b) Show that the set R of all real numbers is a ring under addition and multiplication. — 7

c) Show that if (B, +, . ,/) is a Boolean Algebra, then show that (i) $x + x = x$ and (ii) $(x^{/})^{/} = x$, for every $x \in B$. — 2+3=5

d) Find the Disjunctive Normal Form (DNF) of the Boolean function $f(x, y, z) = y + xz$. — 3

4. a) Prove that in any graph there is an even number of vertices of odd degree. — 5

b) State and prove the Handshaking Lemma — 1+4= 5

c) Use a truth table to determine whether the following arguments form is valid: — 5

$$p \vee q$$
$$p \rightarrow r$$
$$q \rightarrow r$$
$$\therefore r$$

d) Test the validity of the following arguments: — 5

"Some rational numbers are powers of 5. All integers are rational numbers. Therefore some integers are powers of 5"

5. a) Consider the set S= {2, 4, 5, 10, 15, 20} and consider the partial order $\leq$ is the divisibility relation. Then examine whether (S, $\leq$) is poset or not. If S is a poset, then find minimal elements and maximal elements. — 4+3=7

b) Find the number of edges in a complete graph with n vertices. — 4

c) Let G be a simple graph with at least two vertices. Prove that G has at least two vertices of same degree. — 5

d) Define walk, path and trail of a graph with example. — 4

6.	**a)**	Prove that a connected graph with p vertices and (p -1) edges is a tree.	5
	b)	Define tree, forest in graphs. Prove that a tree with n vertices has (n-1) edges.	2+3=5
	c)	Determine the number of edges in a graph with 6 vertices, two of degree 4 and four of degree 2.	3
	d)	Find the maximum number of vertices in a connected graph having 17 edges.	3
	e)	Draw a graph (if it exists) having the following properties or explain why no such graph exists: A graph with 4 edges and 4 vertices having degree 1, 2, 3 and 4.	4

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