

Total No. of printed pages = 8

19/3rd Sem/UMA302

2021

DISCRETE MATHEMATICS

Full Marks – 100

Time – Three hours

The figures in the margin indicate full marks for the questions.

Question No. 1 is compulsory and answer any 4 (Four) questions from the rest.

1. (a) Choose the correct options : $1 \times 5 = 5$
- (i) The number of identity element(s) in a group is
- (a) One (b) two
- (c) infinite (d) None of these
- (ii) If S be the set of people, then the relation R on S defined by xR_y if x is father of y is
- (a) Reflexive (b) Symmetric
- (c) Transitive (d) Not transitive

[Turn over



(iii) If $A = \{4a, 3b, 2c\}$ and $B = \{2a, b, d, 3c\}$ be two multiplicity sets, then $A \cdot B$ is

- (a) $\{4a, 3b, 3c, d\}$ (b) $\{4a, 3b, 3c\}$
(c) $\{4a, 3b, 2c, d\}$ (d) $\{2a, 3b, 3c, d\}$

(iv) The function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = x^2$ is

- (a) One-one (b) Onto
(c) Manyone into (d) One-one onto

(v) In the multiplicative group $\{1, -1, i, -i\}$, the order of i is

- (a) 2 (b) 4
(c) 3 (d) 1

(b) Fill up the blanks : $1 \times 5 = 5$

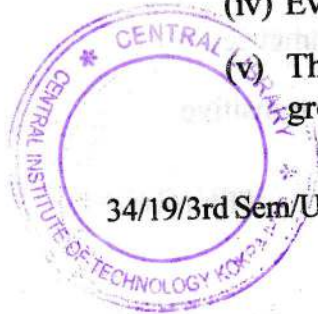
(i) If the function $f : A \rightarrow B$ is invertible, then its inverse is _____.

(ii) A cyclic permutation of length two is called _____.

(iii) The number of elements in the symmetric set of permutation P_3 is _____.

(iv) Every cyclic group is _____.

(v) The order of each sub-group of a finite group is a _____ of the order of the group.



(c) Choose the correct options : $1 \times 10 = 10$

(i) A graph in which all nodes are of equal degree is known as

- (a) A regular graph (b) Multigraph
(c) Planner graph (d) Non-regular graph

(ii) A poset in which every pair of elements has both a least upper bound and a greatest lower bound is termed as

- (a) Sublattice (b) Lattice
(c) Trail (d) Walk

(iii) If every two elements of a poset are comparable then the poset is called

- (a) Sub ordered set (b) Lattice
(c) Trail (d) Walk

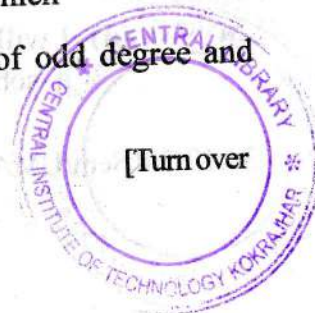
(iv) A _____ has a greatest and a least element which satisfies $0 \leq a \leq 1$ for every 'a' in the lattice (say L)

- (a) Semi lattice (b) Join semi lattice
(c) Meet semi lattice (d) Bounded lattice

(v) A Euler graph is one in which

- (a) only two vertices are of odd degree and rests are even

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- (b) only two vertices are of even degree and rests are odd
- (c) all the vertices are of odd degree
- (d) all the vertices are of even degree
- (vi) How many edges are there in a graph with 20 vertices and the sum of the degrees (in degree and out degree) is 100 ?
- (a) 100 (b) 50
- (c) 40 (d) 20
- (vii) A closed walk in which no vertex (except its terminal vertices) appears more than once is called a/an
- (a) Path (b) Trail
- (c) Circuit (d) Euler circuit
- (viii) The negation of the statement "If I become a teacher, then I will open a school" is
- (a) I will become a teacher and I will not open a school
- (b) Either I will not become a teacher or I will open a school
- (c) Neither I will become a teacher nor I will open a school
- (d) I will not become a teacher or I will open a school



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(4)

(ix) Let $P(n)$ be a statement and let $P(n) \Rightarrow P(n+1)$, for all natural number n , then $P(n)$ is true

- (a) for all natural number n
- (b) for all $n > 1$
- (c) for all $n > m$, m being a fixed positive integer
- (d) Nothing can be said

(x) Translate the given statement into First-order logic.

“For every a , if a is a poet, then a is a writer”

- (a) \exists a poet $((A). \text{writer } ((A)$
- (b) \forall a poet $((A). \text{writer } ((A)$
- (c) All of these.
- (d) None of these.

2. (a) Prove that $A \times (B \cup C) = (A \times B) \cup (A \times C)$ 4

(b) If $f : A \rightarrow B$ and $g : B \rightarrow C$ be two invertible functions then show that $g \circ f$ is also invertible.



(c) If I be the set of all integers, then show that the relation R on I defined by xRy if and only if $x-y$ is divisible by 5, (for all integers x and y) is an equivalence relation. 5

(d) (b) Define the following : 6

(i) Sub-group

(ii) Normal sub-group

(iii) Cyclic group.

3. (a) Define group. Show that the set $G = \{1, w, w^2\}$, where w is an imaginary cube root of unity is a group under multiplication. 6

(b) Show that, a non-empty subset H of a group G is a subgroup of G if and only if

(i) $a \in H, b \in H$ implies $ab \in H$

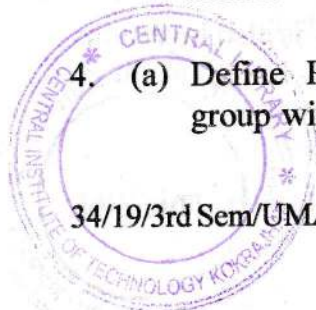
(ii) $a \in H$ implies $a^{-1} \in H$. 2+8=10

(c) Show that if H and K are two normal subgroups of a group G , then $H \cap K$ is also a normal sub-group of G . 6

4. (a) Define Permutation group and Alternating group with examples. 4

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(6)





(b) Determine which of the following are even or odd permutation : 3

(i) $(1\ 2\ 3)(1\ 2)$

(ii) $(1\ 2\ 3\ 4\ 5)(1\ 2\ 3)(4\ 5)$

(iii) $(1\ 2)(1\ 3)(1\ 4)(2\ 5)$.

(c) If for every element $a, b \in G$, where G is a group, then show that G is abelian if

$$(ab)^2 = a^2b^2. \quad 3$$

(d) Define Homomorphism of a group.

Show that the set of all integers is a ring under addition and multiplication. 2+8

5. (a) Use a truth table to determine whether the following arguments form is valid : 7

$$\begin{array}{l} p \vee q \\ p \rightarrow r \\ q \rightarrow r \end{array}$$

$\therefore r$

(b) Test the validity of the following arguments :

“Some rational numbers are powers of 5. All integers are rational numbers. Therefore some integers are powers of 5” 7

(c) Prove by contradiction to show that $\sqrt{2}$ is an irrational number. 6

- 6 (a) Show that every chain is a distributive lattice. 6
- (b) Consider the set $S = \{2, 4, 5, 10, 15, 20\}$ and consider the partial order \leq is the divisibility relation. Then examine whether (S, \leq) is poset or not. If S is a poset, then find minimal elements and maximal elements. 7
- (c) Consider the set $S = \{1, 5, 6, 8, 10\}$ and consider the relations
- $R = \{(1,1), (5, 5), (6, 6), (8, 8), (10, 10), (1, 6), (8, 6), (6, 1)\}$. Draw the digraph of the above relation. Examine whether this relation is anti-symmetric or not. 7
7. (a) Find the number of edges in a complete graph with n vertices. 4
- (b) Let G be a simple graph with at least two vertices. Prove that G has at least two vertices of same degree. 5
- (c) Let G be a simple graph with at most $2n$ vertices. If the degree of each vertex is at least n , then prove that the graph is connected. 5
- (d) Define Walk, Path and Trail of a graph with example. 6

