Total No. of printed pages = 8

19/3rd Sem/UMA302

2021

DISCRETE MATHEMATICS

Full Marks – 100

Time – Three hours

The figures in the margin indicate full marks
for the questions.

Question No. *1 is compulsory* and answer any *4* (*Four*) questions from the rest.

1. (a) Choose the correct options : $1 \times 5 = 5$

(i) The number of identity element(s) in a group is

(a) One (b) two

(c) infinite (d) None of these

(ii) If S be the set of people, then the relation R on S defined by ${}_xR_y$ if x is father of y is

(a) Reflexive (b) Symmetric

(c) Transitive (d) Not transitive

[Turn over

(iii) If A = {4a, 3b, 2c} and B = {2a, b, d, 3c} be two multiplicity sets, then A B is

(a) {4a, 3b, 3c, d} (b) {4a, 3b, 3c}

(c) {4a, 3b, 2c, d} (d) {2a, 3b, 3c, d}

(iv) The function $f : R \rightarrow R$ defined by $f(x) = x^2$ is

(a) One-one (b) Onto

(c) Manyone into (d) One-one onto

(v) In the multiplicative group {1, –1, i, –i}, the order of i is

(a) 2 (b) 4

(c) 3 (d) 1

(b) Fill up the blanks : $1 \times 5 = 5$

(i) If the function $f : A \rightarrow B$ is invertible, then its inverse is ____.

(ii) A cyclic permutation of length two is called ____.

(iii) The number of elements in the symmetric set of permutation P_3 is ____.

(iv) Every cyclic group is ____.

(v) The order of each sub-group of a finite group is a ____ of the order of the group.

(c) Choose the correct options : $1\times10=10$

(i) A graph in which all nodes are of equal degree is known as

(a) A regular graph (b) Multigraph

(c) Planner graph (d) Non-regular graph

(ii) A poset in which every pair of elements has both a least upper bound and a greatest lower bound is termed as

(a) Sublattice (b) Lattice

(c) Trail (d) Walk

(iii) If every two elements of a poset are comparable then the poset is called

(a) Sub ordered set (b) Lattice

(c) Trail (d) Walk

(iv) A ______ has a greatest and a least element which satisfies $0 \leq a \leq 1$ for every 'a' in the lattice (say L)

(a) Semi lattice (b) Join semi lattice

(c) Meet semi lattice (d) Bounded lattice

(v) A Euler graph is one in which

(a) only two vertices are of odd degree and rests are even

(b) only two vertices are of even degree and rests are odd

(c) all the vertices are of odd degree

(d) all the vertices are of even degree

(vi) How many edges are there in a graph with 20 vertices and the sum of the degrees (in degree and out degree) is 100 ?

(a) 100 (b) 50

(c) 40 (d) 20

(vii) A closed work in which no vertex (except its terminal vertices) appears more than once is called a/an

(a) Path (b) Trail

(c) Circuit (d) Euler circuit

(viii) The negation of the statement "If I become a teacher, then I will open a school" is

(a) I will become a teacher and I will not open a school

(b) Either I will not become a teacher or I will open a school

(c) Neither I will become a teacher nor I will open a school

(d) I will not become a teacher or I will open a school

(ix) Let $P(n)$ be a statement and let $P(n) \Rightarrow P(n+1)$, for all natural number n, then $P(n)$ is true

(a) for all natural number n

(b) for all $n > 1$

(c) for all $n > m$, m being a fixed positive integer

(d) Nothing can be said

(x) Translate the given statement into First-order logic.

"For every a, if a is a poet, then a is a writer"

(a) $\exists$ a poet ((A). writer ((A)

(b) $\forall$ a poet ((A). writer ((A)

(c) All of these.

(d) None of these.

2. (a) Prove that $A \times (B \cup C) = (A \times B) \cup (A \times C)$ 4

(b) If $f : A \rightarrow B$ and $g : B \rightarrow C$ be two invertible functions then show that $g_o f$ is also invertible. 5

(c) If I be the set of all integers, then show that the relation R on I defined by ${}_xR_y$ if and only if x–y is divisible by 5, (for all integers x and y) is an equivalence relation. 5

(d) (b) Define the following : 6

(i) Sub-group

(ii) Normal sub-group

(iii) Cyclic group.

3. (a) Define group. Show that the set $G = \{1, w, w^2\}$, where w is an imaginary cube root of unity is a group under multiplication. 6

(b) Show that, a non-empty subset H of a group G is a subgroup of G if and only if

(i) $a \in H, b \in H$ implies $ab \in H$

(ii) $a \in H$ implies $a^{-1} \in H$. 2+8=10

(c) Show that if H and K are two normal sub-groups of a group G, then $H \cap K$ is also a normal sub-group of G. 6

4. (a) Define Permutation group and Alternating group with examples. 4

(b) Determine which of the following are even or odd permutation : 3

(i) (1 2 3)(1 2)

(ii) (1 2 3 4 5)(1 2 3)(4 5)

(iii) (1 2)(1 3)(1 4)(2 5).

(c) If for every element a, b $\in$ G, where G is a group, then show that G is abelian if $(ab)^2 = a^2b^2$. 3

(d) Define Homomorphism of a group.
Show that the set of all integers is a ring under addition and multiplication. 2+8

5. (a) Use a truth table to determine whether the following arguments form is valid : 7

$$p \vee q$$

$$p \rightarrow r$$

$$q \rightarrow r$$

$$\therefore r$$

(b) Test the validity of the following arguments :
"Some rational numbers are powers of 5. All integers are rational numbers. Therefore some integers are powers of 5" 7

(c) Prove by contradiction to show that $\sqrt{2}$ is an irrational number. 6

6 (a) Show that every chain is a distributive lattice. 6

(b) Consider the set S= {2, 4, 5, 10, 15, 20} and consider the partial order $\leq$ is the divisibility relation. Then examine whether (S, $\leq$ is poset or not. If S is a poset, then find minimal elements and maximal elements. 7

(c) Consider the set S = {1, 5, 6, 8, 10} and consider the relations

R = {(1,1), (5, 5), (6, 6), (8, 8), (10, 10), (1, 6), (8, 6), (6, 1)}. Draw the digraph of the above relation. Examine whether this relation is anti-symmetric or not. 7

7. (a) Find the number of edges in a complete graph with n vertices. 4

(b) Let G be a simple graph with at least two vertices. Prove that G has at least two vertices of same degree. 5

(c) Let G be a simple graph with at most 2n vertices. If the degree of each vertex is at least n, then prove that the graph is connected. 5

(d) Define Walk, Path and Trail of a graph with example. 6