UG/3rd/UMA302

2021

DISCRETE MATHEMATICS

Full Marks: 100

Time: Three hours

The figures in the margin indicate full marks for the questions.

Answer Q. No. 1(compulsory) and any four from Q.No.2-7.

1. a) State *True* or *False*: 1 x 10=10

i) The multiplication group {1, -1} is a subgroup of the multiplicative group {1, -1, i, -i}.

ii) If H and K are two normal subgroups of a group G, then their union may be normal subgroup of G.

iii) Let G and K be two groups and let $f: G \to K$ be a group homomorphism. Then Ker f is a normal subgroup of G.

iv) The ring of integers has zero divisors.

v) A field can be defined as a commutative division ring.

vi) The function $f: R \to R$ defined by $f(x) = x^2$ is many one.

vii) If S be the set of people, then the relation R on S defined by xRy if x is father of y is not transitive.

viii) If R and S are two equivalence relations on a set A, then $R \cup S$ may not be an equivalence relation on A.

ix) If $A = \{1,2,3,4\}$, then power set of A contains 15 elements.

x) Compositions of functions is an associative operation.

b) Fill in the blanks: 1 x 5=5

i) If R and S are relations from A to B, then $(R \cap S)^{-1} =$..........

ii) If $f: A \to B$ is one-one onto, then $f^{-1}: B \to A$ is ....

iii) The cardinality of the multi set $\{a, a, b, b, c\}$ is..........

iv) If $A = \{4a, 3b, 2c\}$ and $B = \{2a, b, d, 3c\}$ be the multi sets, then $A \cup B$ is…………

v) If $A = \{a, b, c\}$, then the identity relation on A is ………

c) Choose the correct option: 1x5 = 5

i) A closed walk in which no vertex (except its terminal vertices) appears more than once is called a/an ________________[path/ Eulerian /circuit /trail].

ii) Choose the correct statement:

[A] Every walk is a path [B] Every circuit is a path

[C] Every loop is a circuit [D] The origin and terminal of a walk are always distinct.

iii) If a simple graph has 15 edges, then sum of the degrees of all the vertices is ______. [25/24/50/30]

iv) If a graph G has 7 vertices and 9 edges, then the size of the adjacency matrix is ______[7×7 / 7×9 / 9×9 / None of these]

v) Which of the following is not a logical statement?

[A] Today is Monday [B] There is no rain without cloud

[C] If the diagonals of quadrilateral bisect each other then it is a parallelogram

[D] Square of a real number is always negative.

2. a) Prove that a simple graph with at least two vertices has at least two vertices of same degree. 5

b) Show that the maximum number of edges in a simple graph with p vertices is $\frac{p(p-1)}{2}$. 4

c) Prove that the edge connectivity of a graph G cannot exceed the minimum degree of a vertex in G. 5

d) Prove that a tree T with n-vertices has (n-1) edges. 6

3. a) In a group $(G, *)$ and a, b be any two elements in G and e is the identity element , then prove the followings 6

i) if $b * a = e$, then $a * b = e$

and ii) $a * e = a$ for all $a \in G$

Furthermore, there is only one element $e \in G$ satisfying

iii) for all a ∈ G, there is only one b ∈ G satisfying b ∗ a = e.

	b)	Prove that a finite integral domain is a field.	4
	c)	Show that the multiplicative group G = $\{1, \omega, \omega^2\}$ is a cyclic group. Find all the generator(s) of the cyclic group G.	3+2=5
	d)	Define Integral Domain. Examine whether (Z, +, .) is an integral domain or not.	2+3=5
4.	a)	Give an example of a graph which is Hamiltonian but not Eulerian and vice-versa.	2+2=4
	b)	Define spanning tree. Prove that if there is one and only one path between every pair of vertices in G, then G is a tree.	6
	c)	Find the truth table of the proposition $(\sim p \wedge (\sim q \wedge r)) \vee (q \wedge r) \vee (p \wedge r)$.	5
	d)	Find the principal conjunctive normal form using the truth table of the compound position $(p \wedge q) \vee (\sim q \wedge r)$.	5
5.	a)	Prove that $A \times (B \cup C) = (A \times B) \cup (A \times C)$.	5
	b)	Let $f: A \to B$ and $g: B \to C$ be two invertible functions, then show that gof is also invertible. Also, show that $(gof)^{-1} = f^{-1}og^{-1}$.	10
	c)	If I be the set of all integers, then show that the relation R defined on I by xRy if and only if $x - y$ is divisible by 3, for all $x, y \in I$ is an equivalence relation on I.	5
6.	a)	Prove that the function $f: R \to R$ defined by $f(x) = 2x^3 - 1$ is one-one onto.	5
	b)	If R and S are two equivalence relations on a set A, then show that $R \cap S$ is an equivalence relation on A.	7
	c)	Write all the operations on multi sets.	8