Total number of printed pages: 02 **UG/3rd/UMA301(BACK)**

2024

ENGINEERING MATHEMATICS III

Full Marks: 100

Time: Three hours

The figures in the margin indicate full marks for the questions.

Answer ***any five*** *questions.*

1. Find Laplace Transform of the following functions (any *four*):

(i) t^5e^{-5t} (ii) $tcosat$ (iii) $e^{-\alpha t}cos\beta t$

(iv) t^4e^{3t} (v) t^2sint

5x4=20

2. **a)** Evaluate:

(i) $L^{-1}\left\{\frac{1}{s^2(s+1)^2}\right\}$ (ii) $L^{-1}\left\{\frac{6s-4}{s^2-4s+20}\right\}$

(iii) $L^{-1}\left\{e^{-5s}\frac{1}{(s-2)^4}\right\}$ **5x3=15**

b) If $L\{F(t)\} = f(s)$, then show that $L\{F(at)\} = \frac{1}{a}f\left(\frac{s}{a}\right)$.

OR

Find $Z[\{f(k)\}]$, if $f(k) = \begin{cases} 5^k, & k < 0 \\ 3^k, & k \geq 0 \end{cases}$ **5**

3. **a)** **(i)** When is a complex function $f(z) = u(x, y) + iv(x, y)$ analytic?

(ii) Is the function $f(z) = \cos x \sin y + i \sin x \cos y$ analytic? Justify.

(iii) Determine a, b, c, d so that the function $f(z) = (x^2 + axy + by^2) + i(cx^2 + dxy + y^2)$ is analytic. **2+2+3=7**

b) Show that the function $f(z) = |z|^2$ is differentiable only at the origin. **6**

c) Given that $u = x^2 - y^2$ and $v = 2x - x^3 + 3xy^2$, prove that both u and v are harmonic functions but $u + iv$ is not analytic function of z. **7**

4. **a)** Evaluate the complex integral $\int_C \frac{3z+4}{z(2z+1)}dz$ where C is the circle $|z| = 1$. **6**

b) Determine the poles of $f(z) = \frac{z-3}{(z-2)^2(z+1)}$ and residues at its poles, and hence evaluate $\int_C f(z)dz$ where C is $|z| = 2$. **7**

c) Show that the function $u = x^3 - 2xy - 3xy^2$ is harmonic. Find its harmonic conjugate v and express $f(z)$ in terms of z. **7**

5. **a)** Using Lagrange's Method Solve(any one):

(i) $x^2p + y^2q = (x + y)z$

(ii) $(x^2 - y^2 - z^2)p + 2xyq = 2xz$ **8**

b) Find the complete solution of

(i) $p^2 + q^2 = x^2 + y^2$

(ii) $z^2(p^2 + q^2 + 1) = 1$ **6x2=12**

6. **a)** Solve:

(i) $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = y\cos x$

(ii) $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = x^2 + xy + y^2$ **6x2=12**

b) Using the method of separation of variable solve, $3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 3u$, given that $u(x, 0) = 4e^{-x}$. **8**

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