Total number of printed pages: 03 **UG/Sem III/UMA301**

2023

ENGINEERING MATHEMATICS III

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

Answer ***any five*** *questions.*

1. a) (i) If $L\{F(t)\} = f(s)$, then $L\{F(at)\} =$ ______________ **(1x5=5)**

(ii) $L\{3t^2 + 2t + sint\} =$ ______________

(iii) $L^{-1}\left\{\frac{3s+2}{s^3} - \frac{s-9}{s^2+4}\right\} =$ ______________

(iv) $L\{e^{at} - cost\} =$ ________

(v) If $Z[\{f(k)\}] = F(z)$, then $Z[\{f(k \pm n)\}] =$ ________

b) If $L\{F(t)\} = \frac{e^{-\frac{1}{s}}}{s}$, find $L\{e^{-t}F(3t)\}$. **(5)**

c) Find Laplace transform of the following functions (*Any two*): **(5x2=10)**

(i) $e^{-kt}(Acost + Bsint)$

(ii) $\frac{cosat - cosbt}{t}$

(iii) $t^2 sint$

2. a) Evaluate (*Any two*): **(5x2=10)**

(i) $L^{-1}\left\{\frac{3s+7}{s^2-2s-3}\right\}$

(ii) $L^{-1}\left\{\frac{s}{(s^2+a^2)^2}\right\}$

(iii) $L^{-1}\left\{\frac{6}{2s-3} - \frac{3+4s}{9s^2-16} + \frac{8-6s}{16s^2+9}\right\}$

b) Find $Z[\{f(k)\}]$, if $f(k) = 5^k \quad , k < 0$ **(5)**

$= 3^k \quad , k \geq 0$

c) $Y^{//} + 9Y = cos2t$, given $Y(0) = 1,\ Y\left(\frac{\pi}{2}\right) = -1$ **(5)**

3. a) (i) If $F(u, v) = 0$ and u and v are functions of x, y, z, then find the partial derivative of it w.r.t y. **(1x5=5)**

(ii) Write the Auxiliary equations of Charpit's method.

(iii)What is the special equation name of $Z = px + qy + 2\sqrt{p+q}$?

(iv) Write the n^{th} order linear homogeneous partial differential equation.

(v) Write the complementary function of $\frac{\partial^3 z}{\partial x^3} - 4\frac{\partial^3 z}{\partial x^2 \partial y} + 4\frac{\partial^3 z}{\partial y^3} = 0$.

b) Solve: **(2x5=10)**

(i) $4\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 16 \log(x + 2x)$

(ii) $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = x^2 + xy + y^2$

c) Using the method of separation of variable solve, $3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 3u$, given that $u(x, 0) = 4e^{-x}$ **(5)**

4. **a)** Using direct integration method, solve: $\frac{\partial^3 z}{\partial x^2 \partial y} + 18xy^2 + \sin(2x - y) = 0$ **(3)**

b) Using Lagrange's method Solve : **(7)**

(i) $x^2 p - y^2 q = (x + y)z$

(ii) $\frac{y^2 z}{x} p + xzq = y^2$

c) Find the complete solution of the following (Any one): **(4)**

(i) $p^2 + q^2 = x^2 + y^2$

(ii) $z^2(p^2 + q^2 + 1) = 1$

d) Using Charpits' method solve: $2xz - px^2 - 2qxy + pq = 0$ **(6)**

5. **a)** Answer the following **(1x6=6)**

(i) When is a complex function $f(z) = u(x, y) + iv(x, y)$ analytic?

(ii) Is the function $f(z) = 3x^2 y + i2xy$ analytic? Justify

(iii) If $f(z) = (x + ay) + i(bx + y)$ is analytic then a is equal to ______

(iv) The value of m so that $2x - x^2 + my^2$ may be harmonic is ______

(v) $f(z) = |\bar{z}|^2$ is differentiable only at ________

(vi) The value of $\int_C \frac{z^2+2z}{z+3} dz$, where C is $|z| = 1.5$ is ______

b) Show that the function $f(z) = z^3$ is analytic in the entire z-plane and find $f'(z)$. **(7)**

c) Given that $u = x^2 - y^2 + 2y$ and $v = x^3 - 2xy - 3xy^2$, prove that both u and v are harmonic functions but $u + iv$ is not an analytic function of z. **(7)**

6. **a)** Evaluate the complex integral $\int_C \frac{1-2z}{z(z-1)(z-3)} dz$ where C is the circle $|z| = 2$ **(6)**

b) Determine the poles of $f(z) = \frac{3z^2+z+1}{(z^2-1)(z+3)}$ and residues at its poles, and hence evaluate $\int_C f(z)dz$ where C is $|z| = 2$ **(7)**

c) Define a harmonic function. Show that the function $u = 4xy - 3x + 2$ is harmonic and find its harmonic conjugate. Construct the corresponding analytic function $f(z) = u(x,y) + iv(x,y)$ and express $f(z)$ in terms of z. **(7)**

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