CENTRAL INSTITUTE OF TECHNOLOGY KOKRAJHAR
(Deemed to be University)
KOKRAJHAR :: BTR :: ASSAM :: 783370

END – SEMESTER EXAMINATION
UG

Session: July-December, 2022 | Semester: III | Time: 3 Hrs. | Full Marks: 100

Course Code: UMA301 | Course Title: MATHEMATICS III

Answer question 1 and any four questions from the rest

1. Answer as directed $(20 \times 1 = 20)$

(a) Fill in the blanks

(i) If $f(z) = (x + ay) + i(bx + y)$ is analytic then $a =$ ________

(ii) The harmonic conjugate of $e^y \cos x$ is __________

(iii) The value of m so that $2x - x^2 + my^2$ may be harmonic is ___________

(iv) $f(z) = |\bar{z}|^2$ is differentiable only at ______________

(v) The residue of $f(z) = \frac{1+e^z}{\sin z + z\cos z}$ at the pole $z = 0$ is ________

(vi) The poles of $\frac{z}{\cos z}$ are _________

(vii) The value of $\int_C \frac{z^2+2z}{z+3} dz$, where C is $|z| = 1.5$ is _________

(b) Fill in the blanks

(viii) $L\{(t^3 + 1)^2\} =$ ___________

(ix) $Z[\{a^k\}]$ for $k \geq 0$ is ___________

(x) Z-transform of the unit impulse is __________

(xi) Z-transform of the discrete unit step function is ____________

(xii) $L^{-1}\left\{\frac{s}{s^2+4a^2}\right\} =$ _________

(xiii) $L^{-1}\left\{\frac{3+4s}{9s^2-16}\right\} =$ __________

(c) Answer the following short questions

(xiv) Write the form lagrange's equation?

(xv) Write the Auxiliary equations of Charpit's method.

(xvi) What is the special equation name of $Z = px + qy + 2\sqrt{p+q}$?

(xvii) Write the complete solution of $\sqrt{p} + \sqrt{q} = 1$.

(xviii) Write the n^th order linear homogeneous partial differential equation.

(xix) Write the complementary function of $\frac{\partial^3 z}{\partial x^3} - 4\frac{\partial^3 z}{\partial x^2 \partial y} + 4\frac{\partial^3 z}{\partial y^3} = 0$.

(xx) Write the particular integral of $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = e^{x+2y}$.

2. (a) Show that the function $f(z) = \log z$ is analytic everywhere in the complex plane except at origin and find $f'(z)$. (7)

(b) (i) When is a complex function $f(z) = u(x,y) + iv(x,y)$ analytic?

(ii) Is the function $f(z) = 2xy + i(x^2 - y^2)$ analytic? Justify

(iii) Determine a, b, c, d so that the function

$f(z) = (x^2 + axy + by^2) + i(cx^2 + dxy + y^2)$ is analytic. (1+2+3=6)

(c) Given that $u = e^{-2x} \sin 2y$ and $v = x^3 - 2xy - 3xy^2$, prove that both u and v are harmonic functions but $u + iv$ is not analytic function of z. (7)

3. (a) Evaluate the complex integral $\int_C \frac{z}{z^2-3z+2} dz$ where C is the circle $|z-2| = \frac{1}{2}$. (6)

(b) Determine the poles of $f(z) = \frac{z-3}{(z-2)^2(z+1)}$ and residues at its poles, and hence evaluate $\int_C f(z)dz$ where C is $|z| = 2$. (7)

(c) Define a harmonic function. Show that the function $u = x^3 - 2xy - 3xy^2$ is harmonic and find its harmonic conjugate. (7)

4. (a) Define Z-Transform of a sequence $\{f(k)\}$. Find Z-Transform of the sequence $\{f(k)\}$ where $f(k) = 5^k, \quad k \le 0$

$= 3^k, \quad k \ge 0$ (5)

(b) Find $L\{F(t)\}$, if $F(t) = 0 \; for \; 0 < t < 2$

$= 4 \; for \; t \ge 2.$ (5)

(c) If $L\{F(t)\} = \frac{e^{-\frac{1}{s}}}{s}$, find $L\{e^{-2t}F(3t)\}$. (5)

(d) Evaluate: $L^{-1}\left\{\frac{6s-4}{s^2-4s+20}\right\}$. (5)

5. (a) If $L\{F(t)\} = f(S)$, prove that $L\{e^{at}F(t)\} = f(s-a)$, for $s > a$. (5)

(b) Find Laplace transform of the following functions (any two): (5x2=10)

(i) $e^{at}(1 - t + \frac{t^2}{2})$ (ii) $t \cos at$ (iii) $\frac{Sinht}{t}$

(c) Solve (using Laplace transform): $Y'' + Y = t$, given $Y(0) = 1, \; Y'(0) = 2$. (5)

6. (a) Find the complete solution of (12)

(i) $p^2 + q^2 = x^2 + y^2$

(ii) $z^2(p^2 + q^2 + 1) = 1$

(b) Using Charpit's method solve $2xz - px^2 - 2qxy + pq = 0$ (8)

7. (a) Solve: (12)

(i) $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = y \cos x$

(ii) $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = x^2 + xy + y^2$

(b) Using the method of separation of variable solve

$$3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 3u,$$

given that $u(x, 0) = 4e^{-x}$ (8)