

2021

ENGINEERING MATHEMATICS III

Full Marks: 100

Time: Three hours

The figures in the margin indicate full marks for the questions.

Answer Q. No. 1(compulsory) and any four from Q.No.2-7.

1. a) State True or False:

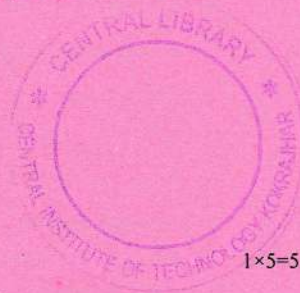
1×10=10

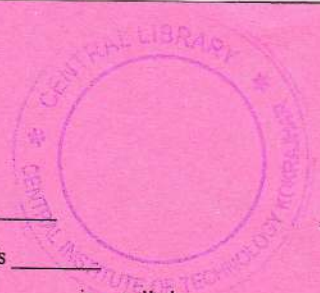
i) A partial differential equation has more than one dependent variable.

ii) Lagrange method is used to form the partial differential equation from:
 $z = xf(x) + yg(y)$.iii) Degree and order of the partial differential equation $p(1+q) = qz$ are one and two respectively.iv) Partial differential equation $z = p^2x + q^2y$ is solvable by Charpit's method.v) Particular integral of $\frac{\partial^2 z}{\partial x^2} + 4\frac{\partial^2 z}{\partial x^2} - 5\frac{\partial^2 z}{\partial y^2} = \sin(2x + 3y)$ is α .vi) Complementary function of $\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = e^{2x+y}$ is $f_1(x+2y) + f_2(x+2y)$.vii) Partial differential equation of $2Z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ is $z = px + qy$.viii) $L\{\sin at\} = \frac{s}{s^2 + a^2}$ ix) $L^{-1}\left\{\frac{1}{s^2}\right\} = t$.x) $L^{-1}\left\{\frac{1}{s-a}\right\} = t^2$.

b) Fill in the blanks:

1×5=5

i) If $f(z) = (x + \alpha y) + i(bx + y)$ is analytic then $\alpha = \dots\dots\dots$ ii) The harmonic conjugate of $e^y \cos x$ is $\dots\dots\dots$ iii) $f(z) = |z|^2$ is differentiable only at $\dots\dots\dots$ 



- iv) The poles of $\frac{z}{\cos z}$ are _____
 - v) The value of $\int_C \frac{z^2 + 2z}{z+3} dz$, where C is $|z| = 1.5$ is _____
 - vi) The residue of $f(z) = \frac{1+e^z}{\sin z + r \cos z}$ at the pole $z = 0$ is _____
 - vii) The integral $\int_0^\infty e^{-st} F(t) dt$, $s \in R$ or C is called _____
 $[L\{F(t)\}$ or $Z\{F(t)\}]$.
 - viii) The property $L\{F(at)\} = \frac{1}{a} f\left(\frac{s}{a}\right)$, where $L\{F(t)\} = f(s)$ is called _____
 (Linear Property/Change of scale property).
 - ix) $L\{2e^{-4t}\} =$ _____
 - x) $L\{3t - 2\} =$ _____
2. a) Solve the linear partial differential equation: $\frac{y^2 z}{x} p + xzq = y^2$ 5
- b) Solve the non-linear partial differential equations: $p(1+q) = qz$ 5
- c) Solve the following linear homogeneous partial differential equations: 2×5=10
- i) $\frac{\partial^2 z}{\partial x^2} - 2 \frac{\partial^2 z}{\partial x^2 \partial y} = 2e^{2x} + 3x^2 y$
 - ii) $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6 \frac{\partial^2 z}{\partial y^2} = 16 \log(x+2y)$
3. a) Using the method of separation of variables, solve $\frac{\partial u}{\partial x} = 4 \frac{\partial u}{\partial y}$, where $u(0, y) = 8e^{-3y}$. 6
- b) Form the partial differential equation of $f(x^2 + y^2, z - xy) = 0$. 4
- c) Solve the equation $\frac{\partial u}{\partial z} = \frac{\partial^2 u}{\partial x^2}$ with the boundary conditions $u(x, 0) = 3 \sin(n\pi x)$, $u(0, t) = 0$ and $u(l, t) = 0$, $0 < x < l, t > 0$. 10
4. a) Show that the function $f(z) = \log z$ is analytic everywhere in the complex plane except at origin and find $f'(z)$. 7
- b) i) When is a complex function $f(z) = u(x, y) + iv(x, y)$ analytic? 1+2+3=6
- ii) Is the function $f(z) = x^2 y + ixy^2$ analytic? Justify
- iii) Determine a, b, c, d so that the function $f(z) = (x^2 + axy + by^2) + i(cx^2 + dxy + y^2)$ is analytic.
- c) Given that $u = x^2 - y^2$ and $v = \frac{-y}{x^2 + y^2}$, prove that both u and v are harmonic functions but $u + iv$ is not analytic function of z . 7
5. a) Evaluate the complex integral $\int_C \frac{z}{z^2 - 3z + 2} dz$ where C is the circle $|z - 2| = \frac{1}{2}$. 6
- b) Determine the poles of $f(z) = \frac{z-1}{(z+1)^2(z-2)}$ and residues at its poles, and hence evaluate $\int_C f(z) dz$ where C is $|z - i| = 2$. 7

- c) (c) Define a harmonic function. Show that the function $u = x^3 - 2xy - 3xy^2$ is harmonic and find its harmonic conjugate. 7
6. a) Find Laplace Transform of the following functions: 5x3=15
- $(t^2 + 1)^3$
 - $e^{-2t} \cos 4t$
 - $t^3 e^{4t}$
- b) Solve (using Laplace Transform): $Y'' + Y = t$, Given that $Y(0) = 1$, $Y'(0) = 2$. 5
7. a) Evaluate (any two): 6.5x2=13
- $L^{-1} \left\{ \frac{6s-4}{s^2-4s+10} \right\}$
 - $L \{ t^2 \sin t \}$
 - $L \left\{ \frac{\sinh t}{t} \right\}$
- b) Evaluate (Any one): 7
- $L^{-1} \left\{ \frac{s}{(s^2+a^2)^2} \right\}$
 - $Z \{ \{ a^{|k|} \} \}$

