Total number of printed pages:3 **UG/3rd/UMA301**

2021

ENGINEERING MATHEMATICS III

Full Marks: 100

Time: Three hours

The figures in the margin indicate full marks for the questions.

*Answer **Q. No. 1(compulsory) and any four from Q.No.2-7.***

1. a) State *True* or *False*: 1×10=10

i) A partial differential equation has more than one dependent variable.

ii) Lagrange method is used to form the partial differential equation from: $z = xf(x) + xg(y)$.

iii) Degree and order of the partial differential equation $p(1+q) = qz$ are one and two respectively.

iv) Partial differential equation $z = p^2x + q^2y$ is solvable by Charpit's method.

v) Particular integral of $\frac{\partial^2 z}{\partial x^2} + 4\frac{\partial^2 z}{\partial x^2} - 5\frac{\partial^2 z}{\partial y^2} = \sin(2x + 3y)$ is $\propto$

vi) Complementary function of $\frac{\partial^3 z}{\partial x^3} - 4\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = e^{2x+y}$ is $f_1(x + 2y) + f_2(x + 2y)$.

vii) Partial differential equation of $2Z = \frac{x^2}{a^2} + \frac{x^2}{b^2}$ is $z = px + qy$.

viii) $L\{\sin at\} = \frac{s}{s^2 + a^2}$

ix) $L^{-1}\left\{\frac{1}{s^2}\right\} = t$.

x) $L^{-1}\left\{\frac{1}{s-a}\right\} = t^2$.

b) Fill in the blanks: 1×5=5

i) If $f(z) = (x + \alpha y) + i(bx + y)$ is analytic then a=.....

ii) The harmonic conjugate of $e^y \cos x$ is _____

iii) $f(z) = |\bar{z}|^2$ is differentiable only at _____

iv) The poles of $\frac{z}{\cos z}$ are _____

v) The value of $\int_C \frac{z^2+2z}{z+3}dz$, where C is $|z| = 1.5$ is _____

vi) The residue of $f(z) = \frac{1+e^z}{\sin z + z\cos z}$ at the pole $z = 0$ is _____

vii) The integral $\int_0^\infty e^{-st}F(t)dt$, $s \in R$ or C is called .....$[L\{F(t)\}$ or $Z[\{F(t)\}]]$.

viii) The property $L\{F(at)\} = \frac{1}{a}f\left(\frac{s}{a}\right)$, where $L\{F(t)\} = f(s)$ is called .....(Linear Property/Change of scale property).

ix) $L\{2e^{-4t}\}$ =.....

x) $L\{3t-2\}$ =.....

2. a) Solve the linear partial differential equation: $\frac{y^2z}{x}p + xzq = y^2$ 5

b) Solve the non-linear partial differential equations: $p(1+q) = qz$ 5

c) Solve the following linear homogeneous partial differential equations: 2×5=10

i) $\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} = 2e^{2x} + 3x^2y$

ii) $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = 16\log(x+2y)$

3. a) Using the method of separation of variables, solve $\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}$, where $u(0,y) = 8e^{-3y}$. 6

b) Form the partial differential equation of $f(x^2+y^2, z-xy) = 0$. 4

c) Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ with the boundary conditions $u(x,0) = 3\sin(n\pi x), u(0,t) = 0$ and $u(l,t) = 0$, $0 < x < 1, t > 0$. 10

4. a) Show that the function $f(z) = \log z$ is analytic everywhere in the complex plane except at origin and find $f'(z)$. 7

b) i) When is a complex function $f(z) = u(x,y) + iv(x,y)$ analytic? 1+2+3=6

ii) Is the function $f(z) = x^2y + ixy^2$ analytic? Justify

iii)Determine a, b, c, d so that the function $f(z) = (x^2 + axy + by^2) + i(cx^2 + dxy + y^2)$ is analytic.

c) Given that $u = x^2 - y^2$ and $v = \frac{-y}{x^2+y^2}$, prove that both u and v are harmonic functions but $u + iv$ is not analytic function of z. 7

5. a) Evaluate the complex integral $\int_C \frac{z}{z^2-3z+2}dz$ where C is the circle $|z-2| = \frac{1}{2}$. 6

b) Determine the poles of $f(z) = \frac{z-1}{(z+1)^2(z-2)}$ and residues at its poles, and hence evaluate $\int_C f(z)dz$ where C is $|z-i| = 2$. 7

c) (c) Define a harmonic function. Show that the function $u = x^3 - 2xy - 3xy^2$ is harmonic and find its harmonic conjugate. 7

6. a) Find Laplace Transform of the following functions: 5x3=15

i) $(t^2+1)^3$

ii) $e^{-2t}cos4t$

iii) t^3e^{4t}

b) Solve (using Laplace Transform): $Y'' + Y = t$, Given that $Y(0) = 1$, $Y'(0) = 2$. 5

7. a) Evaluate(any *two*): 6.5x2=13

i) $L^{-1}\left\{\frac{6s-4}{s^2-4s+20}\right\}$

ii) $L\{t^2 sint\}$

iii) $L\left\{\frac{sinhs}{s}\right\}$

b) Evaluate (Any one): 7

i) $L^{-1}\left\{\frac{s}{(s^2+a^2)^2}\right\}$

ii) $Z[\{a^{|k|}\}]$