Total number of printed pages: 03 **Programme (UG)/2nd/UMA201**

2022

ENGINEERING MATHEMATICS II

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

Answer ***any five*** *questions.*

1. (a) Find the inverse of the following matrix using Cayley-Hamilton Theorem 5

$$\begin{bmatrix} 1 & 3 & 5 \\ 3 & 2 & 4 \\ 5 & 4 & 2 \end{bmatrix}$$

(b) Solve the following system of equations using GE method. 5

$$x + 2y + 3z = 0$$

$$2x + 3y - 2z = 0$$

$$4x + 7y + 4z = 0$$

(c) Reduce the following matrix to row echelon form 5

$$A = \begin{bmatrix} 1 & 3 & 5 \\ 2 & -1 & 4 \\ -2 & 8 & 2 \end{bmatrix}$$

(d) Find the rank of the matrix 5

$$A = \begin{pmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & -1 \\ 10 & 3 & 9 & 7 \end{pmatrix}$$

2. (a) Define rank of a matrix. Find the value of k, if the rank of the matrix 5

$\begin{bmatrix} 1 & 5 & 4 \\ 0 & 3 & 2 \\ k & 13 & 10 \end{bmatrix}$ is 2.

(b) Test the consistency of the following system of equations and if yes, solve them. 5

$$x + 2y - z = 3$$

$$3x - y + 2z = 1$$

$$2x - 2y + 3z = 2$$

$$x - y + z = -1$$

(c) Find the eigenvalues and corresponding eigenvectors of the matrix 5

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{bmatrix}$$

(d) Verify Caley –Hamilton theorem for the matrix 5

$$\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

3. (a) Define 1+2+2 = 5

i) Random Variable

ii) Probability Mass Function

iii) Probability Density Function

(b) The Probability density function of a continuous distribution is $f(x) = K(1 - x)$; $0 \leq x \leq 1$. Find K , E(X) and V(X). 5

(c) A card is drawn from a pack 52 cards. Find the probability of getting either a spade or an ace. 5

(d) Let A and B be two events such that P(A) = $\frac{3}{4}$ and P(B) = $\frac{5}{8}$. Show that

(i)P(A∪ B) $\geq \frac{3}{4}$ and (ii) $\frac{3}{8} \leq (A \cap B) \leq \frac{5}{8}$ 2+3=5

4. (a) Find mean and variance of Poission distribution 3+5=8

(b) Two urns A and B contains 6 white and 4 black balls and 5 white and 5 black balls respectively. One urn is selected at random and a white ball is drawn from it. Find the probability that it has come from the urn A. 5

(c) Evaluate the distribution function of the following: 7

$$f(x) = \begin{cases} \frac{x}{3} & for\ 0 < x \leq 1 \\ \frac{5(4-x)}{27} & for\ 1 < x \leq 4 \\ 0, & otherwise \end{cases}$$

5. (a) Form differential equation from $y = (A + Bx)e^{5x}$, where A and B are arbitrary constants. 5

(b) Solve: 5x3=15

(i) $\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$

(ii) $\frac{dy}{dx} = \frac{2x + 3y + 4}{4x + 6y + 5}$

(iii) $x(x^2 + y^2 - a^2)dx + y(x^2 - y^2 - b^2)dy = 0$

6. Solve: 5x4=20

(a) $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 2e^{3x}$

(b) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 3y = Sinx$

(c) $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = x$

(d) $\frac{d^3y}{dx^3} - 13\frac{dy}{dx} - 12y = 0$

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