2023

ENGINEERING MATHEMATICS II

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

*Answer **any five** questions.*

1. a) Form differential equation from the primitive $xy = ae^x + be^{-x} + x^2$,   5
        where a and b are arbitrary constants.

    b) Solve: 5x3=15

        i) $\dfrac{dy}{dx} = \dfrac{y-x}{x+x}$

        ii) $(x + 2y + 1)dx = (2x + 4y + 3)dy$

        iii) $x(x^2 + y^2 - a^2)dx + y(x^2 - y^2 - b^2)dy = 0$

2. Solve: 5x4=20

    i) $x\dfrac{dy}{dx} + 2y = \dfrac{y^3}{x^2}$

    ii) $\dfrac{d^2y}{dx^2} + y = cos^2x$

    iii) $\dfrac{d^2y}{dx^2} - 4\dfrac{dy}{dx} + 3y = 2e^{3x}$

    iv) $\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} = x^2 + 2x + 4$

3. Solve: 6+7+7=20

    i) $x^2\dfrac{d^2y}{dx^2} - 4x\dfrac{dy}{dx} + 6y = x$

    ii) If $\dfrac{dy}{dx} + 2ytanx = sinx$ and if $y = 0$ when $x = \frac{\pi}{3}$. Express y in terms of x.

iii) $\frac{dx}{dt} - 7x + y = 0$

$\frac{dy}{dt} - 2x - 5y = 0$

4. a) Find mean and variance of Poission distribution.   5+5=10

    b) If X is a random variable with probability density function $f(x) = Ce^{-ax}$ 2+4+4=10
where $o < x < \alpha$, find C, E(X) and V(X).

5. a) Justify the following function as probability density function or not given   5
as follows:

$$f(x) = \begin{cases} x \ for \ 0 < x \leq 1 \\ \dfrac{3-x}{4} \ for \ 1 < x \leq 3 \\ 0, otherwise \end{cases}$$

    b) The chances that doctor C will diagnose disease X correctly is 60 %. The   5
chances that a patient will die by his treatment after correct diagnosis is
40 % and the chances of death by wrong diagnosis is 70 %. A patient of
doctor C, who had disease X died. Find the probability that his disease
was diagnosed correctly.

    c) Show that $V(X) = E(X^2) - \{E(X)\}^2$ 5

    d) Two urns A and B contains 6 white and 4 black balls and 5 white and 5   5
black balls respectively. One urn is selected at random and a white ball is
drawn from it. Find the probability that it has come from the urn A.

6. a) Analyse the following system of simultaneous linear equations by Gauss's   5
Elimination method.

$2x - y + 3z = -2$

$4x + 2y - z = -16$

$6x + 2y + 4z = -2$

    b) Reduce the following matrix to row echelon form   6

$$A = \begin{bmatrix} 1 & 1 & 2 & -5 \\ 2 & 5 & -1 & 9 \\ 2 & 1 & -1 & 3 \\ 1 & 3 & 2 & 7 \end{bmatrix}$$

    c) Find the inverse of the following matrix by using row elementary   5
transformation.

$$X = \begin{bmatrix} 1 & 5 & 2 \\ 1 & 1 & 7 \\ 0 & -3 & 4 \end{bmatrix}$$

d) Consider the three vectors S= {[1, 0, 0], [0, 1, 0], [0, 0, 1]}. Examine whether the set S, of vectors is linearly dependent or not.   4

7. a) Find the eigenvalues and corresponding eigenvectors of the matrix   5+3=8

$$X = \begin{bmatrix} -3 & 1 & -1 \\ -7 & 5 & -1 \\ -6 & 6 & -2 \end{bmatrix}$$

b) Verify the Cayley-Hamilton Theorem for the following matrix and find the inverse of the matrix   5+1=6

$$X = \begin{bmatrix} 1 & 3 & 0 \\ -2 & 2 & -1 \\ 4 & 0 & -2 \end{bmatrix}$$

c) Let $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$. Is $D = \begin{bmatrix} 2 & 8 \\ 4 & 2 \end{bmatrix}$ a linear combination of A, B and C?   3

d) Show that any square complex matrix can be expressible uniquely as sum of a Hermitian and a skew-Hermitian matrix.   3

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