2022

ENGINEERING MATHEMATICS II

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

*Answer **any five** questions.*

1. **a)** Solve the following simultaneous linear equations by Gauss's Elimination method.   **5**
    $$x - y - z = 2$$
    $$3x - 3y + 2z = 16$$
    $$2x - y + z = 9$$

    b) |Reduce the following matrix to row echelon form   **6**
    $$A = \begin{bmatrix} 1 & 2 & -4 & -4 & 5 \\ 2 & 4 & 0 & 0 & 2 \\ 2 & 3 & 2 & 1 & 5 \\ -1 & 1 & 3 & 6 & 5 \end{bmatrix}$$

    c) Show that any square matrix can be expressible uniquely as the sum of a symmetric and a skew-symmetric matrix.   **4**

    d) Find the inverse of the following matrix by using row elementary transformation.   **5**
    $$X = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 2 & 4 \\ 1 & -3 & -3 \end{bmatrix}$$

2. **a)** Consider the three vectors S= {[1, 2, 0], [1, 1, -1], [1, 4, 2]}. Examine whether the set S, of vectors is linearly dependent or not.   **4**

    b) Find the eigenvalues of the matrix   **5**
    $$X = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

    c) Verify the Cayley-Hamilton Theorem for the following matrix and find inverse   **5**

$$X = \begin{bmatrix} 1 & 3 & 0 \\ -2 & 2 & -1 \\ 4 & 0 & -2 \end{bmatrix}$$

d) Let $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$. Is $D = \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}$ a linear combination of A, B and C? **6**

3. a) Form differential equations from the following: **5x2 = 10**
(i) $(A + Bx)e^{5x}$, where A and B are arbitrary constants
(ii) $(A cos x + B sin x)e^x$, where A and B are arbitrary constants

b) Solve: **5x2 = 10**
(i) $(x^2 + y^2)dx + 2xydy = 0$
(ii) $(x^2 - 4xy - 2y^2)dx + (y^2 - 4xy - 2x^2)dy = 0$

4. Solve (any *four*):

(i) $\dfrac{d^2y}{dx^2} - 4\dfrac{dy}{dx} + 3y = 2e^{3x}$ (ii) $\dfrac{d^2y}{dx^2} - 4\dfrac{dy}{dx} + 3y = 2e^{3x}$ **5x4=20**

(iii) $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + 3y = Sinx$ (iv) $x^2 \dfrac{d^2y}{dx^2} + 5x\dfrac{dy}{dx} + 4y = x^4$

(v) $\dfrac{dx}{dt} - 7x + y = 0$
$\dfrac{dy}{dt} - 2x - 5y = 0$

5. a) If P(A) = a and P(B) = b, then show that P(A/B) ≥ (a+b-1)/b. **3**

b) State Baye's theorem. The chances that doctor C will diagnose disease X correctly is 60 %. The chances that a patient will die by his treatment after a correct diagnosis is 40 % and the chances of death by the wrong diagnosis is 70 %. A patient of doctor C, who had disease X died. Find the probability that his disease was diagnosed correctly. **2+5=7**

c) Find mean and variance of Binomial distribution. **4+6=10**

6. a) Define: **1+2+2=5**
(i) Distribution function   (ii) Mathematical Expectation   (iii) Moments

b) If X is a random variable with probability density function $f(x) = Ce^{-ax}$ where $o < x < \propto$, find C, E(X) and V(X). **2+3+3=8**

c) The function is given as follows: **7**

$$f(x) = \begin{cases} x \ for \ 0 < x \le 1 \\ \dfrac{3 - x}{4} \ for \ 1 < x \le 3 \\ 0, otherwise \end{cases}$$

Find the distribution function.

****