Total number of printed pages: 02 **UG/1st /UMA101**

2024

ENGINEERING MATHEMATICS-I

Full Marks: 100

Time: Three hours

The figures in the margin indicate full marks for the questions.

*Answer **any five** questions.*

1. a) Find the centre and radius of curvature of $y = 3x^2 + xy + y^2 - 4x$ at $(0,0)$. 5

   b) What kind of improper integral is $\int_1^2 \frac{x}{x\sqrt{x^2-1}}\,dx$? Examine its convergence. 5

   c) Use Comparison test to examine the convergence of $\int_0^1 \frac{1}{x^{\frac{1}{3}}(1+x^2)}\,dx$ 5

   d) Verify that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$ where $u = \log_e\left(\frac{x^2+y^2}{xy}\right)$ 5

2. a) Show that (i) $\beta\left(\frac{1}{2}, \frac{1}{2}\right) = \left\{\gamma\left(\frac{1}{2}\right)\right\}^2$ and (ii) $\frac{\gamma\left(\frac{7}{2}\right)\gamma\left(\frac{3}{2}\right)}{2\gamma(6)} = \frac{\pi}{128}$ 5

   b) Use Beta & Gamma functions to show that $\int_0^\infty \frac{x}{1+x^6}\,dx = \frac{\pi}{3\sqrt{3}}$ 5

   c) Find the volume of the solid generated by revolving the cardioid $r = a(1 + \cos\theta)$ about the initial line. 5

   d) Find the surface area of the sphere of radius 'a', the equation of the circle being $r = a$. 5

3. a) Verify Rolle's Theorem for $f(x) = x^3 + 3x^2 - 24x - 80$ in $[-4, 5]$ 5

   b) Verify Lagrange's Mean Value Theorem for $f(x) = x(x-1)(x-2)$ in $\left[0, \frac{1}{2}\right]$ 5

   c) Evaluate $\lim_{x\to 0} \frac{\tan x - x}{x^2 \tan x}$ 5

   d) Find the maximum and minimum values of $f(x) = (2x-1)^2 + 3$ 5

4. a) Discuss the continuity at the origin of $f(x,y) = \begin{cases} \frac{1}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$ 5

   b) Find the values of x and y for which $f(x,y) = x^2 + y^2 + 6x - 12$ has a minimum value, Also find this minimum value. 5

   c) Find the first 3 terms in the Maclaurin's series for 5X2=10

(i) $\sin^2 x$ (ii) xe^{-x}

5. Test the Convergence of the following series: 4X5=20

(i) $\sum u_n$, where $u_n = \frac{3^n+1}{4^n+1}$

(ii) $\frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + \cdots \ldots \ldots \ldots \ldots$

(iii) $\frac{2!}{3} + \frac{3!}{3^2} + \frac{4!}{3^3} + \cdots \ldots \ldots \ldots \ldots \ldots$

(iv) $1 + \frac{x}{2} + \frac{x^2}{5} + \frac{x^3}{10} + \cdots \ldots \ldots \ldots$

6. a) Changing the order of the integration evaluate the following integrals: 3X2=6

(i) $\int_0^1 \int_x^{2-x} \frac{x}{y}\, dxdy$

(ii) $\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dxdy$)

b) Find the area that lies inside the cardioid $r = a(1 + cos\theta)$ and outside the circle $r = a$ 4

c) Find the coordinates of the centre of gravity of the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$, the density being given $kxyz$ 6

d) If $r^2 = x^2 + y^2 + z^2$, show that $\nabla f(r) = \frac{f'(r)}{r}\vec{r}$ and hence show that $\nabla \int r^n\, dr = r^{n-1}\vec{r}$ 4

7. a) Find the directional derivative of the scalar point function $\emptyset = 3e^{2x-y+z}$ at the point A(1,1,-1) in the direction towards the point B(-3,5,6) 5

b) Find $div\, \vec{V}$ *and* $cul\, \vec{V}$ *if* $\vec{V} = \nabla(x^3 + y^3 + z^3 + 3xyz)$ 4

c) Find the line integral, $\int_c (x^2 + y^2)dx + (y + 2x)dy$, where C is the closed boundary of the region in the first quadrant, that is bounded by the curves $y^2 = x$ *and* $x^2 = y$ 5

d) Verify the Green's lemma in the xy-plane for $\oint_C (xy + y^2)dx + x^2dy$, where C is the closed curve of the region bounded by $y = x$ *and* $y = x^2$ 6

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