Total number of printed pages:2 **B.Tech /Sem I/UMA101**

2023

ENGINEERING MATHEMATICS I

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

Answer ***any five*** *questions.*

1. a) Find y_n of (i) $y = \frac{x}{x-1}$ (ii) $y = e^x x^3$ $2 + 3 = 5$

   b) If $y = e^{\tan^{-1} x}$ then show that 5

   $$(1 + x^2)y_{n+2} + [(2n + 1)x - 1]y_{n+1} + n(n + 1)y_n = 0$$

   c) Prove that the curves $\frac{x^2}{a} + \frac{y^2}{b} = 1$ and $\frac{x^2}{c} + \frac{y^2}{d} = 1$ will intersect orthogonally if $a + d = b + c$. 5

   d) (i) Calculate the arc length of $y = x^2$ between $x = 0$ and $x = 2$ 5

   (ii) Show that for the rectangular hyperbola $xy = c^2$, the radius of curvature, $\rho = \frac{(x^2+y^2)^{\frac{3}{2}}}{2c^2}$

2. a) Use Euler's Theorem on homogeneous function to show that if 5

   $$u = log\left(\frac{x^4+y^4}{x+y}\right) \text{ then } x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3.$$

   b) (i) Evaluate $\int_0^{\pi/2} \cos^8 \theta \, d\theta$ 2+3 = 5

   (ii) If $y = x + \tan x$, show that $cos^2 x \frac{d^2y}{dx^2} - 2y + 2x = 0$

   c) Prove by Taylor's Theorem that 5

   $$\tan\left(\frac{\pi}{4} + x\right) = 1 + 2x + 2x^2 + \frac{8}{3}x^3 + \frac{10}{3}x^4 + \cdots$$

   d) Show that $\beta(m, n) = \beta(m + 1, n) + \beta(m, n + 1)$ 5

3. a) Show that the series $\frac{1}{3} + \left(\frac{2}{5}\right)^2 + \left(\frac{3}{7}\right)^3 + \cdots \ldots \ldots$ is convergent 6

b) Expand $f(x) = 2x + 1$ as a half-range Fourier Sine series in $0 < x < 2\pi$. 6

c) Expand $f(x) = x^2$ as a Fourier series in $0 < x < 2$. 8

4. a) Test the following series: 6+6 =12

(i) $x + \frac{x^2}{2} + \frac{x^3}{3} + \cdots \ldots \ldots$ (ii) $\sum_{n==1}^{\propto} \frac{2n}{3^n}$

b) Find the Fourier series for the function $\begin{cases} x & for -\pi < x < 0 \\ -x & for \quad 0 < x < \pi \end{cases}$ 8

5. a) Answer the following short questions: 3x3 = 9

i) Prove that $Curl(\emptyset\vec{F}) = (grad\emptyset) \times \vec{F} + \emptyset Curl\, \vec{F}$

ii) Show that $\vec{F} = (2xy + z^3)\hat{\imath} + x^2\hat{\jmath} + 3z^2x\,\hat{k}$ is a conservative field.

iii) A fluid motion given by $\vec{V} = (y+z)\hat{\imath} + (z+x)\hat{\jmath} + (x+y)\hat{k}$. Is this motion irrotational?

b) If $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ prove that $Curl(r^n\vec{r}) = 0$ 4

c) Use the Gauss divergence theorem to evaluate $\iint_S \vec{F}.d\vec{s}$, where $\vec{F} = x^3\hat{\imath} + y^3\hat{\jmath} + z^3\hat{k}$ and s surface of the sphere $x^2 + y^2 + z^2 = 4$ 7

6. a) If $\vec{V} = (x^2 - y^2 + 2xz)\hat{\imath} + (xz - xy - yz)\hat{\jmath} + (z^2 + x^2)\hat{k}$ is a vector field, find $curl\ \hat{V}$. Show that the vectors given by $curl\ \hat{V}$ at $P_0(1,2,-3)$ and $P_1(2,3,-2)$ are orthogonal. 5

b) Evaluate the line integral $\oint_C (x^2 + y^2)dx + (y + 2x)dy$, where C is the boundary of the region in the first quadrant, that is bounded by the curves $y^2 = x \ and \ x^2 = y$. 8

c) Using the Green's lemma to evaluate the line integral $\int (x^2 + y^2)dx + xydy$ taken over $x^2 + y^2 = 4, x \geq 0, y \geq 0$ 7

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