2023

ENGINEERING MATHEMATICS-I

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

Question no.1 is compulsory and answer ***any four*** *from the questions no 2-7*

1. a) Fill in the blanks 1 x 7=7

(i) If $\sqrt{x}+\sqrt{y}=5$ then y_2 equals ________

(ii) The equation of the tangent line to the curve $y=2x\sin x$ at the point $(\frac{\pi}{2},\pi)$ is ________

(iii) The angle of intersection of the curves $y=x^2$ and $6y=7-3x$ at $(1,1)$ is ________

(iv) The length of the subnormal at the point (1,3) of the curve $y=x^2+x+1$ is ________

(v) The distance covered by a particle moving along a curve $y=\frac{2}{3}x^{\frac{3}{2}}$ from the point $(0,0)$ to the point $\left(4,\frac{16}{3}\right)$ is ________

(vi) The partial derivative of the function $f(x,y)=e^{1-x\cos y}+5e^{\left(\frac{-1}{1+y^2}\right)}$ with respect to x at the point $(1,0)$ is ____________

(vii) $\int_0^{\infty} e^{-x}x^6\,dx$ equals ________

b) State true or false 1 x 5=5

(i) The series $\sum_{n==1}^{\infty}\frac{1}{n}$ is convergent.

(ii) The series $\sum_{n=1}^{\infty} r^n$ is convergent if $r<1$.

(iii) If $\sum x_n$ and $\sum y_n$ are two positive term series such that $\lim_{n\to\infty}\frac{x_n}{y_n}=a\ (\neq 0\ and\ finite)$, then both the series $\sum x_n$ and $\sum y_n$ are converge or diverge together.

(iv) The value of $\lim_{n\to\infty}\left(1+\frac{1}{\sqrt{n}}\right)^{\sqrt{n}}$ is e.

(v) In a Fourier series of an odd function in the interval (-l, l), where l is any real number, the Fourier coefficient $b_n = 0$.

c) (i) If $\emptyset = x^2 + y^2$, find curl(div $\emptyset$) 1

(ii) Write the vector identity of $Curl(\emptyset\vec{F})$ 1

(iii) What is the curl of the vector field $2x^2y\hat{\imath} + 5z^2\hat{\jmath} - 4yz\hat{k}$ 2

(iv) Write the statement of Green's theorem 2

(v) Find the value of $\oint_C (\cos(x)\sin(y) - y)dx + (\sin(x)\cos(y))dy$, where c is the unit circle. 2

2. a) Find y_n of (i) $y = \frac{x}{x-1}$ (ii) $y = e^x x^3$ 2+3

b) If $y = \log(x + \sqrt{1+x^2})$ then show that 5

$(1+x^2)y_{n+2} + (2n+1)xy_{n+1} + n^2y_n = 0$

c) Prove that the curves $\frac{x^2}{a} + \frac{y^2}{b} = 1$ and $\frac{x^2}{c} + \frac{y^2}{d} = 1$ will intersect orthogonally if $a + d = b + c$. 5

d) (i) Calculate the arc length of the polar curve 5

$$r = 1 + \sin\theta;\ 0 \le \theta \le \frac{\pi}{2}.$$

(ii) (ii) Find the radius of curvature of the parabola $y^2 = 16x$ at the end of its latus rectum.

3. a) Use Euler's Theorem on homogeneous function to show that if 5

$u = tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$.

b) (i) Evaluate $\int_0^{\pi/2} \cos^8\theta\, d\theta$ 2+3

(ii) If $ax^2 + 2hxy + by^2 = 1$, show that $\frac{d^2y}{dx^2} = \frac{h^2-ab}{(hx+by)^3}$

c) Prove by Taylor's Theorem that 5

$$\tan\left(\frac{\pi}{4} + x\right) = 1 + 2x + 2x^2 + \frac{8}{3}x^3 + \frac{10}{3}x^4 + \cdots$$

d) Show that $\beta(m,n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}dx$ 5

4. a) Write the statement of the following: 1+2+2

(i) P-test

(ii) Cauchy's Root test

(iii) Fourier series

b) Show that the series $\frac{1}{3} + \left(\frac{2}{5}\right)^2 + \left(\frac{3}{7}\right)^3 + \cdots \ldots\ldots$ is convergent 5

c) Expand $f(x) = x$ as a Fourier series in $0 < x < 2$. 5

d) Examine the convergency of the series $\frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + \cdots \ldots \ldots \ldots$ 5

5. a) Find the Fourier series for the function $\begin{cases} x & for -\pi < x < 0 \\ -x & for \quad 0 < x < \pi \end{cases}$ 10

b) Test the series $\sum_{n=1}^{\infty} \left(\frac{n^{n^2}}{(n+1)^{n^2}}\right)$ 5

c) Expand $f(x) = 2x - 1$ as a half-range Fourier cosine series in $0 < x < 1$. 5

6. a) If $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ and $\vec{a}$ is a constant vector, show that $curl(\vec{a} \times \vec{r}) = 2\vec{a}$ 4

b) If $\vec{V} = (x^2 - y^2 + 2xz)\hat{\imath} + (xz - xy - yz)\hat{\jmath} + (z^2 + x^2)\hat{k}$ is a vector field, find curl($\vec{V}$). Show that the vectors given by curl($\vec{V}$) at $P_0(1,2,-3)$ and $P_1(2,3,-2)$ are orthogonal. 6

c) Verify Green's Lemma for $\oint_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$, where C is the boundary of the area enclosed by the curves $y = \sqrt{x}$ and $y = x^2$ 10

7. a) Find $\iint_S \vec{F}.d\vec{S}$, where $\vec{F} = (2x + 3z)\hat{\imath} - (xz + y)\hat{\jmath} + (y^2 + 2z)\hat{k}$ and s is the surface of the sphere having center at $(3, -1, 2)$ and radius 3. 10

b) Use the Gauss divergence theorem to evaluate $\iint_S (\vec{F}.\hat{n})dS$, where $\vec{F} = x^2z\hat{\imath} + y\hat{\jmath} - xz^2\hat{k}$ and S is the boundary of the region bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 4y$ 10

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