Total No. of printed pages = 5

19/4th Sem/UMA 401

2022

NUMERICAL METHODS AND COMPUTER PROGRAMMING

Full Marks – 100

Time – Three hours

The figures in the margin indicate full marks for the questions.

Answer any *five* questions.

1. (a) Show that $\Delta = E - 1$. 2

(b) Find the missing terms in the following data : 2

x :	1	1.5	2	2.5
y :	6	?	10	20

(c) Find the value of $\nabla^2(4)$ from the following data : 2

x :	0	1	2	3	4	5
y :	4	8	15	7	6	2

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(d) Given $\frac{dy}{dx} = 1 + x - y^2$, $y(0) = 1$, $h = 0.1$, find the difference in the value of y obtained by Improved Euler Method and Euler Method at $x = 0.1$. 3

(e) Given that $\frac{dy}{dx} = x + x^2y$, with $y(0) = 1$ taking $h = 0.1$, find out if there is any improvement in the value of y for $x = 0.1$ obtained by Runge Kutta Fourth Order Method from that of Runge Kutta Third Order. 5

(f) In Regula-Falsi method, x_1 and x_2 are three two initial guess values and x_3 is the 3rd approximation to the root. Then the 4th approximation to the root is given by : 2

$$x_4 = \frac{x_2 f(x_3) - x_3 f(x_2)}{f(x_3) - f(x_2)}$$. State True or False with justification.

(g) Show how the Secant Formula is obtained from the Newton-Raphson Formula for approximating the root of a transcendental equation. 2

(h) Write the condition for convergence of the Method of Successive Approximations. When does the Newton-Raphson method fail ? 2

2. (a) Using Newton's forward interpolation find the value of f(4) for the following data : 6

x :	4	6	8	10
f(x) :	1	3	8	16

(b) Using Newton's Backward interpolation formula, find the value of f(10) for the following data : 6

x :	3	4	5	6	7	8	9
y :	4.8	8.4	14.5	23.6	36.2	52.8	73.9

(c) Using Newton-Cote formula derive the Trapezoidal rule. 8

3. (a) Apply Lagrange's method to find the polynomial f(x) for the given data : 8

x :	5	6	9	11
f(x) :	12	13	14	16

Hence evaluate f(3).

(b) Find $y'(0)$ and $y''(0)$ from the following table : 6

x :	0	1	2	3	4	5
y :	4	8	15	7	6	2

(c) Use Simpson's 1/3rd and 3/8th rule to evaluate $\int_0^{.96} xe^{x^2} dx$ and compare the results with actual value. 6

4. (a) Using Modified Euler method, find y at $x = 0.2$ $x = 0.4$ and given that $\frac{dy}{dx} = y - x^2$, $y(0) = 1$ with correct result upto four places of decimals. 4+4=8

(b) Using Runge-Kutta method of Fourth order, solve $\frac{dy}{dx} = 3x + \frac{1}{2}y$ with $y(0) = 1$ at $x = 0.1$, 0.2, 0.3. 12

5. (a) Using Euler's method, find a solution of $\frac{dy}{dx} = x^2 + y$, with initial condition $y = 1$ at $x = 0$ for the range $0 \le x \le 0.8$ in steps of 0.2. Compare the result with analytical solution. 8+4=12

(b) Solve $\frac{dy}{dx} = 1 + xy^2$ with $y(0) = 1$ for $x = 0.4$ and $x = 0.5$ by Milne's method given that $y(0.1) = 1.105$, $y(0.2) = 1.223$, $y(0.3) = 1.355$. 8

6. (a) Find a real root of $x^3 + 4x + 1 = 0$ correct to three places of decimal using the Bisection Method. 7

(b) Find a real root of $x^3 + 2x - 1 = 0$ correct to three places of decimal using the Successive Approximation Method. 7

(c) Find a real root of $e^{-x} - x = 0$ correct to three places of decimal using the Secant Method. 6

7. (a) Solve the following system of equations using Gauss Elimination Method : 7

$2x_1 + 6x_2 - x_3 = 23,\ 4x_1 - x_2 + 3x_3 = 9,$

$3x_1 + x_2 + 2x_3 = 13.$

(b) Solve the following system of equations using Gauss-Siedal Method : 8

$9x_1 + 2x_2 + 4x_3 = 20,\ x_1 + 10x_2 + 4x_3 = 6,$

$2x_1 - 4x_2 + 10x_3 = -15.$

(c) Find a real root of $e^{-x} - 2x + 1 = 0$ correct to three places of decimal using the Newton Raphson Method. 5