Total number of printed page = 5

19/2nd Sem/UMA 201

2022

ENGINEERING MATHEMATICS - II

Full Marks – 100

Time – Three hours

The figures in the margin indicate full marks for the questions.

Answer any *five* questions.

1. (a) Solve the following simultaneous linear equations by Gauss's Elimination method : 5

$x - y - z = 2$

$3x - 3y + 2z = 16$

$2x - y + z = 9.$

(b) Reduce the following matrix to row echelon form 6

$$A = \begin{bmatrix} 1 & 2 & -4 & -4 & 5 \\ 2 & 4 & 0 & 0 & 2 \\ 2 & 3 & 2 & 1 & 5 \\ -1 & 1 & 3 & 6 & 5 \end{bmatrix}$$

[Turn over

(c) Show that any square matrix can be expressible uniquely as the sum of a symmetric and a skew-symmetric matrix. 4

(d) Find the inverse of the following matrix by using row elementary transformation : 5

$$X=\begin{bmatrix} 1 & 2 & -1 \\ 2 & 2 & 4 \\ 1 & -3 & -3 \end{bmatrix}$$

2. (a) Consider the three vectors S +{[1, 2, 0], [1, 1, –1], [1, 4, 2]}. Examine whether the set S, of vectors is linearly dependent or not. 4

(b) Find the eigenvalues of the matrix : 5

$$X=\begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

(c) Verify the Cayley-Hamilton Theorem for the following matrix and find inverse. 5

$$X=\begin{bmatrix} 1 & 3 & 0 \\ -2 & 2 & -1 \\ 4 & 0 & -2 \end{bmatrix}$$

(d) Let $A=\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, $B=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $C=\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$.

Is $D=\begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}$ a linear combination of A, B and C ? 6

3. Form differential equations from the following : $5\times2=10$

(i) $(A + Bx)e^{5x}$, where A and B are abritrary constants.

(ii) $(A\cos x + B\sin x)e^{x}$, where A and B are arbitrary constants.

(b) Solve : $5\times2=10$

(i) $(x^2 + y^2)dx + 2xydy = 0$.

(ii) $(x^2 - 4xy - 2y^2)dx + (y^2 - 4xy - 2x^2)dy = 0$.

4. Solve (any *four*) : $5\times4=20$

(i) $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 2e^{3x}$

(ii) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 3y = \sin x$

(iii) $x^2 \frac{d^2y}{dx^2} + 5x\frac{dy}{dx} + 4y = x^4$

(iv) $\frac{dx}{dt} - 7x + y = 0$

(v) $\frac{dy}{dt} - 2x - 5y = 0$.

5. (a) If P(A) = a and P(B) = b, then show that $P(A/B) \geq (a+b-1)/b$. 3

(b) State Baye's theorem. The chances that doctor C will diagnose disease X correctly is 60 %. The chances that a patient will die by his treatment after a correct diagnosis is 40 % and the chances of death by the wrong diagnosis is 70 %. A patient of doctor C, who had disease X died. Find the probability that his disease was diagnosed correctly. 2+5=7

(c) Find mean and variance of Binomial distribution. 4+6=10

6. (a) Define : 1+2+2=5

(i) Distribution function

(ii) Mathematical Expectation

(iii) Moments.

(b) If X is a random variable with probability density function $f(x) = Ce^{-ax}$ where $0 < x < \alpha$, find C, E(X) and V(X). 2+3+3=8

(c) The function is given as follows : 7

$$f(x)=\begin{cases} x \text{ for } 0<x\leq 1 \\ \frac{3-x}{4} \text{ for } 1<x\leq 3 \\ 0, \text{ otherwise} \end{cases}$$

Find the distribution function.