Total number of printed pages:4 UG/1st/UMA101

2021

ENGINEERING MATHEMATICS I

Full Marks: 100

Time: Three hours

The figures in the margin indicate full marks for the questions.

Answer Question 1 and any four from the rest of the questions.

1. Fill in the blanks (20x1=20)

(a) If $y = x^4 \log x$ then y_n when $n \geq 5$ is ________

(b) The point on the curve $y = x^3 - 11x + 5$ at which the tangent has the equation $y = x - 11$ is ________

(c) The angle of intersection of the curves $y = x^2$ and $y^2 = x$ at the point (1, 1) is ________

(d) For the curve $by^2 = (a + x)^3$, the square of the subtangent varies as the length of ________ (subnormal/normal/tangent/none).

(e) The length of the arc in one period (t= 0 to $t = 2\pi$) of the cycloid $x = a(t - \sin t), y = a(1 - \cos t)$ is ________

(f) The series $\sum \frac{1}{n^2}$ is ________

(g) The positive term series $1 + r + r^2 + \cdots \ldots \ldots + r^{n-1} + \cdots$ is convergent if ________

(h) The value of $\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$ is ________

(i) If $\sum x_n$ be a positive term series such that $\lim_{n \to \infty} (x_n)^{\frac{1}{n}} = l$, then $\sum x_n$ is convergent if ________

(j) The series $\sum \frac{1}{(2n-1)^p}$ converges if ________

(k) The mathematical perception of the gradient is said to be ________ (tangent/chord/slope/arc).

(l) Curl is defined as the angular velocity at every point of the vector field ________ (True/False).

(m) Surface integral is used to compute ________ (surface/area/volume/density).

(n) The Stoke's theorem uses ________ (divergence/gradient/curl/laplacian).

(o) The divergence of the vector F= $xe^{-x}i + yj - xzk$ is ______

(p) The divergence of distance vector is ________

(q) The projection of A on B, given A = 10j + 3k and B = 4j + 5k is ________

(r) The work done of vector force F and distance d, separated by angle θ can be calculated using ________ (cross product/dot product/addition of two vectors/cannot be calculated)

(s) $\int_0^\infty e^{-x}x^6 dx$ equals ________

(t) $\int_0^\infty e^{-ay}y^{n-1} dy$ equals ________

2. (a) Find y_n of (i) $y = \frac{a-x}{a+x}$ (ii) $y = x^5 \sin x$ (5)

(b) If $y = (sin^{-1}x)^2$ then show that (5)
(i) $(1 - x^2)y_2 - xy_1 - 2 = 0$
(ii) $(1 - x^2)y_{n+2} - 2(n + 1)xy_{n+1} - n^2 y_n = 0$

(c) Prove that the sum of the intercepts of the tangent at any point (x, y) to the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ upon the coordinate axes is constant. (5)

(d) (i) Calculate the arc length of the polar curve
$r = 1 + \sin\theta; 0 \leq \theta \leq \frac{\pi}{2}$.
(ii) Find the radius of curvature of the parabola
$y^2 = 4x$ at the vertex. (5)

3. (a) Use Euler's Theorem on homogeneous function to show that if
$u = sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$. (5)

(b) Find the centre of curvature of $x^3 + y^3 = 2$ at (1,1). (2)

(c) If $ax^2 + 2hxy + by^2 = 1$, show that $\frac{d^2y}{dx^2} = \frac{h^2-ab}{(hx+by)^3}$ (3)

(d) Show that $\tan x = x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \cdots$ (5)

(e) Show that $\beta(m,n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}dx$ (5)

4. (a) Write the statement of (4x2=8)
   (i) Limit Comparison Test
   (ii) Ratio Test
   (iii) Cauchy's Root Test
   (iv) P-test

(b) Test the convergence of the series
$\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \cdots$ (6)

(c) Test the series $\sum_{n=1}^{\infty} \frac{2n-1}{n!}$ (6)

5. (a) Find the angle between the curves $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point (2, 1, 2). (5)

(b) If vectors A and B are irrotational, then show that A × B is solenoidal. (3)

(c) If $\varphi = xyz$ and $A = 2yzi - x^2yj + xz^2k$ and $B = x^2i + yzj - xyk$, find the values of
   (i) $A.\nabla\varphi$
   (ii) $A \times \nabla\varphi$
   (iii) $(A.\nabla)B$ (2×3=6)

(d) If $\emptyset = 2z^2y - xy^2$, find $\nabla\emptyset$ and the directional derivative of $\emptyset$ at (2, 1, 1) in the direction of 3i +6j+ 2k. (1+5=6)

6. (a) Evaluate $\int_C [(x^2 - xy^3)\,dx + (y^2 - 2xy)dy]$, where C is the square with vertices (0, 0), (2, 0), (2, 2), and (0, 2). (6)

(b) If $\vec{F} = (e^z, 1, xe^z)$ then find $\nabla \times \vec{F}$ (3)

(c) Compute $\iint_S FdS$, (6)
where $F = (3x + z, y^2 - \sin(x^2 z), xz + ye^{x^5})$,
and $0 \le x < 1, 0 \le y \le 3, 0 \le z \le 2$.

(d) What is the physical interpretation of curl? (3)

(e) Find the angle between the vectors
$\vec{a} = 3\hat{\imath} + 4\hat{\jmath}$ and $\vec{b} = 6\hat{\imath} - 3\hat{\jmath} + 2\hat{k}$ (2)

7. (a) Find the Fourier series of the function
$$f(x) = \begin{cases} \pi, & 0 < x \le 1 \\ -\pi, & 1 < x < 2 \end{cases}$$ (10)

(b) Prove that a vector function $\vec{f}$ of a scalar variable 't' has constant magnitude if and only if $\vec{f}.\frac{d\vec{f}}{dt} = 0$ (5)

(c) Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ (5)