CENTRAL INSTITUTE OF TECHNOLOGY, KOKRAJHAR
(Centrally Funded Institute under MoE, Govt. of India)
KOKRAJHAR :: B.T.C. :: ASSAM :: 783370

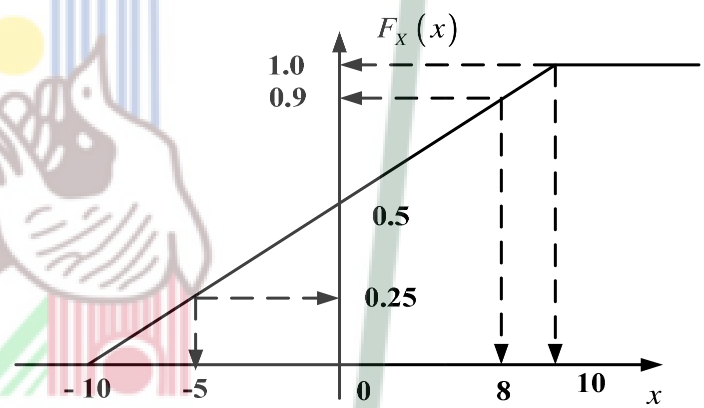
END – SEMESTER EXAMINATION (DEGREE)

Semester: **8th** | Time: **3 Hrs** | Full Marks: **100**
Course Code: **UECE812A** | Course Title: **Information Theory and Coding**

Answer **any five** questions

1) **a**) Define a discrete memory less source (DMS). What is meant by one bit of information? Show that the average information content of the DMS per symbol is given by: $H(S)=\sum_{i=0}^{M-1} p_i \times \log_2\left(\frac{1}{p_i}\right)$; where '$p_i$' is the probability of the occurrence of the RV 'x_i' of a random process **S**. (**2+2+6**)

b) The probability distribution function of a certain RV 'X' is shown below:



From the above distribution, calculate the following probabilities:
a) $P(X>-5)$ b) $P(-5<X\le 8)$ c) $P(X>0)$
Hence calculate and draw the corresponding probability distribution function $f_X(x)$. (**8+2**)

2) **a**) Consider the transformation of a RV '**Y**' by assuming the RV '**X**' has a pdf $f_X(x)=e^{-x}\times u(x)$; where '$u(x)$' is a unit step Heaviside function. If the relation between the RVs '**Y**' and '**X**' is $Y=X^3$; show that the pdf of '**Y**' is given by $f_Y(y)=\frac{e^{-\sqrt[3]{y}}}{3}\times\sqrt[3]{\frac{1}{y^2}}\times u(y)$.

b) Deduce the necessary theory for the above problem. (**4+6**)

3) **a**) If the pdf of the two random variables $(X \text{ and } Y)$ are '$p_X(x)$' and '$p_Y(y)$' respectively, then calculate the pdf '$p_Z(z)$' of the sum of two statistically independent random variables (Z); where $Z=X+Y$. Hence, using the result, show that the Gaussian random variables are 'reproducible', i.e., the sum of two Gaussian random variables is also Gaussian. (**5+5**)

b) A source produces three symbols 'A, B and C' with the following marginal and conditional probabilities:

i	$p(i)$
A	1/4
B	1/4
C	1/2

$p(j\|i)$	**A**	**B**	**C**
A	1/8	1/4	5/8
B	1/2	1/8	3/8
C	3/8	5/8	0

(i) Assume that there is no inter-symbol interference; calculate the entropy of the source.
(ii) Calculate the conditional entropy $H(Y|X)$; where $i \to X$ and $j \to Y$. **(4+6)**

4) **a**) Show that the entropy of an extended DMS (S^n) is given by $H(S^n) = n \times H(S)$; where 'n' is the number of symbols in each block of the DMS with an alphabet size 'M'. A DMS has an alphabet $\{S_0, S_1\}$ with marginal probabilities $p(S_0) = p_0 = \frac{1}{4}$ and $p(S_1) = p_1 = \frac{3}{4}$. Find the entropies of the source 'S' and the extended source 'S^3'; hence show that $H(S^3) = 3 \times H(S)$.
(5+5)

b) Calculate the maximum entropy $(H_{\max})$ of a DMS generating random variables of alphabet size 'M' and show that $H_{\max} = \log_2(M)$ when the random variables are equi-probable. Hence calculate the maximum entropy of a binary source. **(5+2+3)**

5) **a**) What is a binary symmetric channel (BSC). For a BSC, find the channel matrix of the translational probabilities $\left(p\left(y_j|x_i\right)\right)$. A BSC has an error probability of $p = 0.2$, the a-priori probabilities of a logic – O and logic - 1 at the input are 0.4 and 0.6 respectively. What is the probability of receiving a logic - 1 at the output of the BSC? **(2+3+5)**

b) What is channel capacity (C_S)? Hence show that the channel capacity of a BSC is given by $C_S = \left[1 - H(p)\right]$ bits/symbol; where '$H(p)$' is the entropy of a binary DMS. **(2+8)**

6) **a**) Show that the differential entropy of a continuous random variable 'X' is given by $h(X) = \int_{-\infty}^{\infty} f_X(x) \times \log_2\left[\frac{1}{f_X(x)}\right] dx$; where the symbols have their usual meaning. **(10)**

b) Show that the differential entropy of a continuous random variable 'X' is a maximum when it has a Gaussian distribution and this maximum value is given by $h(X) = \frac{1}{2} \times \log_2\left(2\pi e \sigma^2\right)$; where '$\sigma^2$' is the noise variance of the channel. **(10)**

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