Total number of printed pages: 2 **UG/8th/UECE812A**

2023

INFORMATION THEORY AND CODING

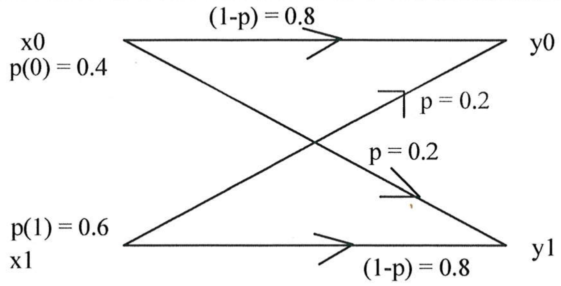
Full Marks: 100

Time: Three hours

The figures in the margin indicate full marks for the questions.

*Answer **any five** questions.*

<table>
<tr><td>1</td><td>a</td><td>Define discrete memory less source (DMS). Show that the average information content of the DMS is given by: $H(s)=\sum_{i=0}^{M-1} p_i \times \log_2 \frac{1}{p_i}$; where '$p_i$' is the probability of the occurrence of the symbol 'x_i' in the symbol set X in a particular sampling instant 'T_S'.</td><td>2+8</td></tr>
<tr><td></td><td>b</td><td>Show that the entropy of an extended DMS (S^n) is given by $H(S^n)=n\times H(S)$; where 'n' is the number of symbols in each block of the DMS with an alphabet size 'M'. A DMS 'S' has an alphabet $\{S_0, S_1\}$ with probability $p(S_0)=p_0=\frac{1}{4}$ and $p(S_1)=p_1=\frac{3}{4}$. Find the entropies of the source 'S' and the extended source 'S^3'.</td><td>5+5</td></tr>
<tr><td>2</td><td>a</td><td>A source produces three symbols 'A, B and C' with the following marginal and conditional probabilities:
<table>
<tr><th>i</th><th>$p(i)$</th></tr>
<tr><td>A</td><td>1/4</td></tr>
<tr><td>B</td><td>1/4</td></tr>
<tr><td>C</td><td>1/2</td></tr>
</table>
<table>
<tr><th>$p(j|i)$</th><th>A</th><th>B</th><th>C</th></tr>
<tr><td>A</td><td>1/8</td><td>1/4</td><td>5/8</td></tr>
<tr><td>B</td><td>1/2</td><td>1/8</td><td>3/8</td></tr>
<tr><td>C</td><td>3/8</td><td>5/8</td><td>0</td></tr>
</table>
(i) Assume that there is no inter-symbol interference; calculate the entropy of the source.
(ii) Calculate the conditional entropy $H(Y|X)$.</td><td>4+6</td></tr>
<tr><td></td><td>b</td><td>What is a binary symmetric channel (BSC)? Find the channel matrix for a BSC.</td><td>2+3+5</td></tr>
</table>

		 The BSC shown above has an error probability of $p = 0.2$. The a-priori probabilities of a '0' and a '1' at the input are 0.4 and 0.6 respectively, shown above. What is the probability of receiving a '1' at the receiving end?	
3	a	Show that the mutual information between the transmitted symbol 'x_i' and the received symbol 'y_j' is given by $I(x_i\|y_j)=\log_2\left[\frac{p(x_i\|y_j)}{p(x_i)}\right]$. Hence show that the average mutual information of the channel is given by $I(X;Y)=H(X)-H(X\|Y)$. Represent this as a Venn diagram.	4+4+2
	b	State Shannon's channel coding theorem. Show that for a discrete memory-less source (DMS) with entropy of '$H(s)$' bits/sym and symbol rate of $\left(1/T_s\right)$ sym/s, the condition for transmitting these symbols to the destination with arbitrary small error probability (P_e) is $\frac{H(s)}{T_s}\leq\frac{C_s}{T_c}$, where the units in both the sides are in bits/s and 'C_s' is the channel capacity of the discrete memory-less channel (DMC) with symbol rate $\left(1/T_c\right)$ sym/s. How this theorem will be modified for a binary symmetric channel (BSC)?	2+7+1
4	a	Prove that when the random variable 'X' corresponds to a Gaussian distribution having mean 'a' and probability density function '$f_X(x)=\frac{1}{\sigma\times 2\pi}\times\exp\left[-\frac{(x-a)^2}{2\sigma^2}\right]$', the differential entropy will be given by $h(X)=\frac{1}{2}\log_2\left(2\pi e\sigma^2\right)$; where '$\sigma^2$' is the variance of the Gaussian process.	12
	b	Show that the differential entropy '$h(X)$' of a continuous random variable 'X' is a maximum when it has a Gaussian distribution with a maximum value of $\frac{1}{2}\log_2\left(2\pi e\sigma^2\right)$.	8
5	a	Discuss Shannon's information capacity theorem. Hence prove that the channel capacity of a band-limited, power-limited Gaussian channel with input power 'S' is given by	4+8

		$C = B \times \log_2\left(1+\frac{S}{N}\right)$ bits/s, where the symbols have their usual meaning.	
	b	An analog signal having 4 kHz bandwidth is sampled at 1.25 times the Nyquist rate and each sample is quantized into one of 256 equally likely levels. Assume that the successive samples are statistically independent, (a) What is the information rate of this source? (b) Can the output of this source be transmitted without error over an AWGN channel with a bandwidth of 10 kHz and an (S/N) ratio of 20 dB? (c) Find the (S/N) ratio required for error-free communication for part (a). (d) Find the bandwidth required for an AWGN channel for error-free communication of the output of this source if the (S/N) ratio is 20 dB.	2+2+2+2
6	a	What is an ideal system? Calculate the Shannon's limit for the ideal system. Hence, discuss 'power-limited' and 'band-limited' operation of a Gaussian channel.	1+4+7
	b	In a certain picture transmission, there are 2.2×10^6 picture elements/frame. For good reproduction, 12 brightness levels are necessary. Assuming all the levels are equally likely, calculate the channel bandwidth without coding to transmit one picture frame every three minutes. Assume the (S/N) ratio over the channel to be 30 dB.	8

XXXXXXXX