Total number of printed pages: 2 **UG/8th/UECE812A**

2024

Information theory and Coding

Full Marks: 100

Time: Three hours

The figures in the margin indicate full marks for the questions.

*Answer **any five** questions.*

1.	a)	Define a discrete memory less source (DMS). What is meant by the entropy of a DMS? Show that the average information content of the DMS is given by: $H(S)=\sum_{i=0}^{M-1} p_i \times \log_2\left(\frac{1}{p_i}\right)$; where 'p_i' is the probability of the occurrence of the RV 'x_i' of a random process **S**.	2+2+6
	b)	Consider the transformation of a RV '**Y**' by assuming the RV '**X**' has a pdf $f_X(x)=e^{-x}\times u(x)$; where '$u(x)$' is a unit step Heaviside function. If the relation between the RVs '**Y**' and '**X**' is $Y=X^3$; show that the pdf of '**Y**' is given by $f_Y(y)=\frac{e^{-\sqrt[3]{y}}}{3}\times\sqrt[3]{\frac{1}{y^2}}\times u(y)$.	10
2.	a)	b) Prove that the pdf of the sum of two statistically independent RV $(W=X+Y)$ is the convolution of their individual RVs.	10
	b)	Show that the mutual information between the transmitted symbol 'x_i' and the received symbol 'y_j' is given by $I(x_i\|y_j)=\log_2\left[\frac{p(x_i\|y_j)}{p(x_i)}\right]$. Hence show that the average mutual information of the channel is given by $I(X;Y)=H(X)-H(X\|Y)$; where the symbols have their usual meaning.	4+6
3.	a)	Show that the entropy of an extended DMS (S^n) is given by $H(S^n)=n\times H(S)$; where 'n' is the number of symbols in each block of the DMS with an alphabet size 'M'. A DMS has an alphabet $\{S_0,S_1\}$ with marginal probabilities $p(S_0)=p_0=\frac{1}{4}$ and $p(S_1)=p_1=\frac{3}{4}$. Find the entropies of the source 'S' and the extended source 'S^3'.	5+5
	b)	A source produces three symbols 'A, B and C' with the following marginal and conditional probabilities:	3+7

i	$p(i)$
A	1/4
B	1/4
C	1/2

$p(j\|i)$	**A**	**B**	**C**
A	1/8	1/4	5/8
B	1/2	1/8	3/8
C	3/8	5/8	0

(i) Assume that there is no inter-symbol interference; calculate the entropy of the source.

(ii) Calculate the conditional entropy $H(Y|X)$

4.	a)	Prove that when the random variable 'x' corresponds to a Gaussian distribution having mean 'a' and probability density function '$f_X(x)=\frac{1}{\sigma\sqrt{2\pi}}\times\exp\left[-\frac{(x-a)^2}{2\sigma^2}\right]$' the differential entropy will be given by '$h(X)=\frac{1}{2}\log_2\left(2\pi e\sigma^2\right)$', where '$\sigma^2$' is the variance of the Gaussian process 'X'.	10
	b)	Show that the differential entropy '$h(X)$' of a continuous random variable is a maximum when the probability density function of the random variable is Gaussian, and the maximum value of the differential entropy is given by '$h(X)=\frac{1}{2}\log_2\left(2\pi e\sigma^2\right)$', where the symbols have their usual meaning.	10
5.	a)	Discuss Shannon's information capacity theorem. Hence, prove that the channel capacity of a band-limited, power-limited Gaussian channel with input power 'S' is given by $C=B\times\log_2\left(1+\frac{S}{N}\right)$ bits/sec, and the symbols have their usual meaning.	4+10
	b)	What is an ideal information system? Calculate Shannon's limit for an ideal system.	6
6.	Write short notes on **any two** from the following:		10+10
	a)	Average mutual information is always positive $\left(I(X;Y)\geq 0\right)$	
	b)	Binary symmetric channel (BSC) and transitional probability.	
	c)	Channel capacity of a BSC: $C_S\big\|_{BSC}=\left[1-H(p)\right]$ bits/sym.	
	d)	Optimum distribution of input power.	

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