Total number of printed pages: **Programme(UG)/UECE516C**

2023

OPTIMIZATION THEORY

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

Answer Q-8 any four questions from the rest.

1.	a)	State the difference between constrained and unconstrained optimizations	2
	b)	Define convex and concave function. State whether the following functions are convex or concave. Identify its maxima and minima. a) $f(x)=4x^3-48x+10$ b) $f(x)=2-3x-4x^2$	8
	c)	Consider the following problem $f(x)=x^2+54/x$ Assume initial search interval (0,5). Apply Golden section search method for 3 iterations and find the solutions.	10
2.	a)	Find the stationary points of the function: $f(x_1,x_2)=2x_1^3-3x_1^2-6x_1x_2(x_1-x_2-1)$ i) Which of these points are local minima, which are local maxima, and which are neither. ii) How many minimum exist along the joining $[0,1]^T$ and $[1,2]^T$	10
	b)	What are the conditions of convexity of a second order function at a given point. State whether the following functions are convex on the given points For $f(x1,x2)=x_1^4+x_2^3-2x_1^2x_2+4$	6
	c)	What are the difference between Direct search method and Gradient-based methods. Name any two direct search methods and any three gradient-based methods for multi-variable optimization	4
3	a)	Find whether the given direction s at the point x is descent for the	6

		respective function: For $f(x_1,x_2)=2x_1^2+x_2^2-2x_1x_2^2+10x_1/x_2^2, at (i) s=[1,1]^T, (ii) x=[2,3]$	
	b)	State Kuhn-Tucker (KT) condition for Non-linear programming	2
	c)	Write down KT conditions for the following $Maximize: 3x_1^2-2x_2$ subject to $2x_1+x_2=4, x_1^2+x_2^2\leq 19.4$, x1≥0 Find out whether points $(0,4)^T$ and $(3.4,-2.8)^T$ are KT points or not.	12
4.	a)	Describe Random Search algorithm for the optimization of a NLP problem.	6
	b)	Consider the following NLP problem $f(x_1,x_2)=(x_1-3)^2+(x_2-4)^2$ subject to $g_1(x)=26-(x_1-5)^2+x_2^2\geq 0$ $x_1,x_2\geq 0$ Assume initial point $(3,3)^T$ and the initial interval is $(6,6)^T$. Assume other required parameters as your choice. Continue the above problem for consecutive 3 iterations.	14
5.	a)	What do you mean by Linearization of a non linear function at a point. Show with example in $f(x_1,x_2)=(x_1-3)^2+(x_2-4)^2$ at $(4,5)^T$	3
	b)	Discuss Frank-Wolfe optimization algorithm	5
	c)	Linearize the following NLP problem at $x=(1,2)^T$ Minimize $f(x)=(x_1^2+x_2-11)^2+(x_1+x_2^2-7)^2$ subject to $g_1(x) = 26-(x_1-5)^2-x_2^2\geq 0,$ $g_2(x) = 20-4x_1-x_2\geq 0,$ $x_1,x_2\geq 0.$	5

	d)	Solve the above problem using Frank Wolfe's method for 1 iteration. Assume a suitable value for the necessary user defined parameters.	7
6.	a)	Write any five comparative statements between the traditional optimization and evolutionary algorithm.	5
	b)	Classify which of the following algorithms are traditional and evolutionary algorithms i) Gray-Wolf optimization, ii) Levenberg-Marquardt optimization iii) Biogeography-based optimization iv) Gradient-descent optimization	2
	c)	State the Key-steps of Genetic algorithm. Give suitable examples for explaining each of the steps.	10
	d)	Discuss the differences between Roulette Wheel selection and Rank selection.	3
7.	Write short notes(Any two)		10x2
	a)	Particle Swarm Optimization	
	b)	Simulated Annealing	
	c)	Conjugate gradient Algorithm	
	d)	Parallel subspace property for the quadratic optimization.	
8.	A)	Select the correct alternatives	10x1
	a)	An inflection point means i) A minimum ii) A maximum iii) Either a maximum or a minimum iv) Neither a maximum nor a minimum	
	b)	A contour plot i) is 2D projection of higher dimensional function values ii) A closed curve formed by the equation iii) Results equal function function values on a contour line.	
		iv) All of the above	
	c)	Golden section reduces search range by a factor of …….. in each iteration i) 0.5 ii) 0.618 iii) 0.382 iv) either 0.618 or 0.382	
	d)	Newton's method fails I) when the number of variable is more than one	

		ii) if the problem is a maximization problem iii) When Hessian matrix can not be estimated iv) When the Hessian matrix can not be inverted.	
	e)	The Gausse's method I) Does not involve Hessian matrix for optimization ii) Approximately estimate the Hessian matrix with the gradient vectors iii) There is no requirement of the Hessian matrix iv) None of the above	
	f)	An unidirectional search is used for i) Searching a mimimum or maximum in a one variable function ii) Searching the nearest mimimum or maximum to a point. iii) searching the minimum value at a particular direction. iv) None of the above.	
	g)	Hessian is a..... i) A scalor number ii) A vector iii) A rectangular matrix iv) A square matrix	
	h)	Generally the cooling schedule in simulated annealing is i) Linearly increasing ii) Linearly decreasing iii) Exponentially increasing iv) Exponentially decreasing	
	i)	Mutation probability in Genetic Algorithm is kept i) Very small ii) Nearly equal to one iii) A random variable between 0 and 1. iv) Decreases gradually over iterations.	
	j)	The inertia parameter in the velocity equation of PSO is kept at I) Very small ii) Nearly equal to one	

		iii) A random variable between 0 and 1 iv) A random variable between -1 and 1.	
	B)	Answer these short answer types questions	
	a)	Find out the Hessian matrix at $(2,2)^T$ $f(x_1, x_2) = x_1^3 + x_1^2 + (x_2 - 4)^3$	2
	b)	Is the above Hessian matrix positive definite?	1
	c)	Find the value of α using along the direction $(1,2)^T$ for minimizing the following function $f(x_1, x_2) = (x_1 - 4)^2 + (x_2 - 5)^2$	3
	d)	Find out the most elite solution in the population in Genetic algorithm for minimizing the following function $f(x_1, x_2) = (x_1 - 4)^2 + (x_2 - 5)^2$ i) $(2,2)^T$ ii) $(-2,2)^T$ iii) $(4,3)^T$ iv) $(3,5)^T$	2
	e)	What do you mean by conjugate direction. How it is different from descent direction?	2