

2021

**OPRIMIZATION THEORY**

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

Answer Q-1 and any four questions.

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|----|----|---|--------|
| 1. | A  | Answer the correct alternatives   | (10x1) |
|    | a) | A point x is said to be an .....point if the function value locally increases as x increases and decreases as x decreases<br>i)Local optima ii) Global optima iii) inflection point iv) None of the above   |        |
|    | b) | Which of the following method is not a gradient based optimization.<br>i) Exhaustive search ii) golden section search iii) region elimination iv) None of the above is gradient based   |        |
|    | c) | The lower and upper bounds of a variable is a and b respectively. In interval halving method, the error of the function value after n iteration will be<br>i) $(0.5)^{(n/2)}(b-a)$ ii) $0.326^{(n/2)}(b-a)$ iii) $0.618^{(n/2)}(b-a)$ iv) $0.326^{(-n/2)}(b-a)$ |        |
|    | d) | An increasing function has<br>a) negative slope b) positive slope c) second order derivative can be either positive or negative. d) second order derivative is positive always.   |        |
|    | e) | The determinant of the Hessina matrix $H= \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$ is<br>i)12 ii) 0 iii) 20 iv) 16   |        |
|    | f) | The H matrix shown in the above question is<br>i) positive definite ii) negative definite iii) positive semi-definite iv) can not be stated   |        |

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|   | g) In a contour plot smaller the radius is<br>i) smaller the function value ii) higher the function value iii) nearer to the minimum iv) none of the above  |     |
|   | h) An unidirectional search from a point A (2,2) along the direction (3,4) reaches at the point (2.6,2.8) after one iteration. Find the value of $\alpha$ . Where $\alpha$ is the step size.<br>i) 1      ii) .5      iii) .25      iv) .2                            |     |
|   | i) Minima of the following function lies in<br>$f(x, y) = (x-10)^2 + (y-20)^2$<br>a) (1,2)    b) (10,20)    iii) (0,0)    iv) (-10,-20)   |     |
|   | j) Here is an optimization problems<br>Maximize $f(x) = 2x_1 + 3x_2$<br>subject to $x_1 \leq 6$<br>$x_1 + x_2 \geq 2$<br>$x_1, x_2 \geq 0$<br>No of equality and inequality constraints are..... and ....<br>i) 2 and 1    ii) 3 and 0    iii) 3 and 1    iv) 4 and 0 |     |
| B | Answer the short questions  | 5x2 |
|   | i) Build the Hessian matrix at $x_1=2, x_2=3$<br>$f(x_1, x_2) = 10(x_1-20)^2 + 10(x_2-10)^2$  |     |
|   | ii) Find out the minima of the following one dimensional functions<br>$f(x) = x^2 + 54/x$   |     |
|   | iii) Find the cross over between the following parents. Assume single point crossover at position 5<br>p1=1 0 0 0 1 0 1 0 = 146<br>p2=1 0 1 0 0 0 1 1 = 163   |     |
|   | iv) Let an $l$ bit schema can contains 0, 1 and *. Find the maximum possible number schemata.   |     |
|   | v) Linearize the following function at point $x=2$<br>$f(x) = 3x^2 - 2x + 4$  |     |
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| 2  | a) | State the algorithm of Golden section search method.  | 5   |
|    | b) | Use 3 iterations of golden section search method in order to maximize the function in the interval (-5,5)<br>$f(x) = 10 + x^3 - 2x - 5\exp(x)$  | 10  |
|    | c) | Perform 2 iterations of Newton-Raphson's method to find the minima of the following function. Assume initial point $x^{(1)} = 1$<br>$f(x) = x^2 - 54/x$   | 5   |
| 3. | a) | State and differentiate between parallel subspace property and extended parallel subspace property of Powell's Conjugate direction method for the optimization of quadratic functions.  | 10  |
|    | b) | What do you understand by "Decent direction"? Find the optimum of the following function using Newton's method for one iteration. Assume initial point $x^{(0)} = (0,0)$<br>$f(x_1, x_2) = (x_1^2 + x_2 - 11)^2 + (x_1 + x_2^2 - 7)^2$                                    | 2+8 |
| 4) | a) | State what is KKT condition for constrained optimization?   | 5   |
|    | b) | Identify whether the points to the following NLP problem:<br>i) (0,0)<br>ii) (0.1, 0.1)<br>iii) (2,1)<br>are feasible optimal points or not?<br>$Minimize x_1^2 + x_2^2 - 10x_1 + 4x_2 + 2$<br>subject to<br>$x_1^2 + x_2 - 6 \leq 0,$<br>$x_2 \geq x_1,$<br>$x_1 \geq 0$ | 3x5 |
| 5) | b) | State what is the linearized search technique for constrained optimization. Write the algorithm of Frank Wolfe's method for optimization  | 3+7 |
|    | c) | Solve this linear programming problem (2 iterations)<br>Maximize $f(x) = 2x_1 + 3x_2$<br>subject to<br>$x_1 \leq 6$<br>$x_1 + 2x_2 \leq 10$   | 10  |

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|    |      | $x_1 + x_2 \geq 2$ $(x_1, x_2) \geq 0$  |         |
| 6. | a)   | a) State the advantage and disadvantage of non-traditional optimization over traditional optimization. Name five non traditional optimization methods   | 2.5+2.5 |
|    |      | <p>Consider the following multivariable function minimization problems</p> $\text{Minimize } x_1^2 + x_2^2 - 10x_1 + 4x_2 + 2$ $7 \geq (x_1, x_2) \geq -8$ <p>consider a 4 bit solutions for <math>x_1, x_2</math> using binary Genetic algorithm.</p> <p>Assume initial random population has been created as</p> <p>(0,0)</p> <p>(3,2)</p> <p>(5,4)</p> <p>(-3,4)</p> <p>(4,2)</p> <p>(-2,-4)</p> |         |
|    | i)   | Calculate fitness function of the population  | 6       |
|    | ii)  | Apply Roulette wheel selection and select 4 individuals from the population for the mating pool.  | 4       |
|    | iii) | Apply a suitable cross over operation. Apply crossover location 2 and 6. Find the new childrens and the latest population.  | 4       |
|    | iv)  | Apply mutation to the best 2 solutions. Choose mutation points 3 and 7. Find mutated solutions. Discuss whether there is any improvement in the population.   | 3       |
|    | v)   | Find the optimum solution and best fitness value achieved .   | 3       |
| 7. |      | Write short notes( Any two)   | 10x2    |
|    | a)   | Levenberg-Marquardt Algorithm   |         |
|    | b)   | Particle Swarm Optimization   |         |
|    | c)   | Simulated Annealing   |         |
|    | d)   | Lagrangian Duality Theory   |         |