Total No. of printed pages = 9

19/5th Sem/UECE516C

2021

OPRIMIZATION THEORY

Full Marks – 100

Time – Three hours

The figures in the margin indicate full marks for the questions.

Answer question No.1 and any *four* questions.

1. A. Answer the correct alternatives : 1×10=10

(a) A point x is said to be an _____ point if the function value locally increases as x increases and decreases as x decreases

(i) Local optima

(ii) Global optima

(iii) Inflection point

(iv) None of the above

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(b) Which of the following method is not a gradient based optimization ?

(i) Exhaustive search

(ii) Golden section search

(iii) Region elimination

(iv) None of the above is gradient based

(c) The lower and upper bounds of a variable is a and b respectively. In interval halving method, the error of the function value after n iteration will be

(i) $(0.5)^{(n/2)}(b-a)$

(ii) $0.326^{(n/2)}(b-a)$

(iii) $0.618^{(n/2)}(b-a)$

(iv) $0.326^{(-n/2)}(b-a)$

(d) An increasing function has

(i) negative slope

(ii) positive slope

(iii) second order derivative can be either positive or negative

(iv) second order derivative is positive always

(e) The determinant of the Hessina matrix $H=\begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$ is

(i) 12 (ii) 0

(iii) 20 (iv) 16

(f) The H matrix shown in the above question is

(i) positive definite

(ii) negative definite

(iii) positive semi-definite

(iv) can not be stated

(g) In a contour plot smaller the radius is

(i) smaller the function value

(ii) higher the function value

(iii) nearer to the minimum

(iv) None of the above

(h) An unidirectional search from a point A (2,2) along the direction (3,4) reaches at the point (2.6, 2.8) after one iteration. Find the value of α. Where α is the step size.

(i) 1 (ii) .5

(ii) .25 (iv) .2

(i) Minima of the following function lies in $f(x, y) = (x - 10)^2 + (y-20)^2$

(i) (1,2) (ii) (10,20)

(iii) (0,0) (iv) (–10,–20)

(j) Here is an optimization problems

Maximize $f(x) = 2x_1 + 3x_2$

subject to $x_1 \leq 6$

$x_1 + x_2 \geq 2$

$x_1, x_2 \geq 0$

No of equality and inequality constraints are _____ and _____

(i) 2 and 1 (ii) 3 and 0

(iii) 3 and 1 (iv) 4 and 0

B. Answer the short questions : 5×2=10

(i) Build the Hessian matrix at $x_1 = 2$, $x_2 = 3$

$f(x_1, x_2) = 10(x_1 - 20)^2 + 10(x_2 - 10)^2$

(ii) Find out the minima of the following one dimensional functions :

$f(x) = x^2 + 54/x$

(iii) Find the cross over between the following parents. Assume single point crossover at position 5

p1 = 1 0 0 0 1 0 1 0 = 146

p2 = 1 0 1 0 0 0 1 1 = 163

(iv) Let an 1 bit schema can contains 0, 1 and *. Find the maximum possible number schemata.

(v) Linearize the following function at point x=2

$f(x) = 3x^2 - 2x + 4$

2. (a) State the algorithm of Golden section search method. 5

(b) Use 3 iterations of golden section search method in order to maximize the function in the interval (–5, 5)

$f(x) = 10 + x^3 - 2x - 5\exp(x)$ 10

(c) Perform 2 iterations of Newton-Raphson's method to find the minima of the following function. Assume initial point $x^{(1)}=1$

$f(x) = x^2 - 54/x$ 5

3. (a) State and differentiate between parallel subspace property and extended parallel subspace property of Powel's Conjugate direction method for the optimization of quadratic functions. 10

(b) What do you understand by "Decent direction" ? Find the optimum of the following function using Newton's method for one iteration. Assume initial point $x^{(0)} = (0, 0)$ 2+8=10

$f(x_1, x_2) = (x_1^2 + x_2 - 11)^2 + (x_1 + x_2^2 - 7)^2$

4. (a) State what is KKT condition for constrained optimization ? 5

(b) Identify whether the points to the following NLP problem : 3×5=15

(i) (0,0)

(ii) (0.1,0.1)

(iii) (2,1) are feasible optimal points or not ?

Minimize $x_1^2 + x^2_2 - 10\,x_1 + 4\,x_2 + 2$

subject to

$$x_1^2 + x_2 - 6 \le 0,$$

$$x_2 \ge x_1$$

$$x_1 \ge 0.$$

5. (b) State what is the linearized search technique for constrained optimization. Write the algorithm of Frank Wolfe's method for optimization. 3+7=10

(c) Solve this linear programming problem (2 iterations) 10

Maximize $f(x) = 2x_1 + 3x_2$

subject to

$$x_1 \le 6$$

$$x_1 + 2x_2 \le 10$$

$$x_1 + x_2 \ge 2$$

$$(x_1, x_2) \ge 0$$

6. (a) State the advantage and disadvantage of non-traditional optimization over traditional optimization. Name five non traditional optimization methods.

Consider the following multivariable function minimization problems

Minimize $x_1^2 + x_2^2 - 10x_1 + 4x_2 + 2$

$7 \geq (x_1, x_2) \geq -8$

Consider a 4 bit solutions for x_1, x_2 using binary genetic algorithm.

Assume initial random population has been created as

(0, 0)	(3, 2)
(5, 4)	(–3, 4)
(4, 2)	(–2, –4)

(i) Calculate fitness function of the population. 6

(ii) Apply Roulette wheel selection and select 4 individuals from the population for the matting pool. 4

(iii) Apply a suitable crossover operation. Apply crossover location 2 and 6. Find the new childrens and the latest population. 4

(iv) Apply mutation to the best 2 solutions. Choose mutation points 3 and 7. Find mutated solutions. Discuss whether there is any improvement in the population. 3

(v) Find the optimum solution and best fitness value achieved. 3

7. Write short notes on any *two* : 10×2=20

(a) Levenberg-Marquardt Algorithm

(b) Particle Swarm Optimization

(c) Simulated Annealing

(d) Lagrangian Duality Theory.