Total number of printed pages: **UG/5th /UECE516C**

2022

Optimization Theory

Full Marks : 100

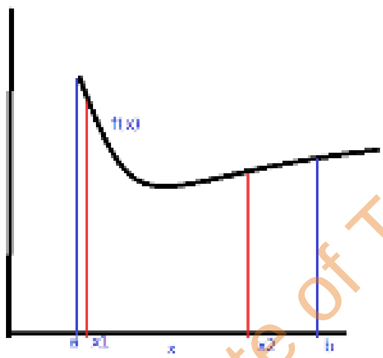
Time : Three hours

The figures in the margin indicate full marks for the questions.

Answer ***Q-1 and any four*** *questions from the rest.*

1.	A	Answer any ten questions.	10x1
	a)	A convex function is guaranteed to have ………….. i) One Minimum ii) One Maximum iii) More than one minimum iv) More than one maxima	
	b)	The contour plots of a parabolic hypersurface is i) Concentric circles ii) Concentric ellipses iii) concentric rectangles iv) None of the above	
	c)	Which of the following optimization method is not a gradient based method? I) Golden section search ii) Secant's method iii) Levenbarg-Marquardt algorithm iv) Newton's method	
	d)	A function is given as $f(x) = x^2 + 54/x$ I) f(x) has a minima at x=3 ii) f(x) has a maxima at x=3	

		iii) f(x) has a maxima in x=-3 iv) f(x) has a minima at x=-3.	
	e)	A simplex search method needs a minimum of ……points to for an N variable optimization i) N ii) N+1 iii) N-1 iv) 2N	
	f)	An unidirectional search is used for i) Searching a mimimum or maximum in a one variable function ii) Searching the nearest mimimum or maximum to a point. Iii) searching the minimum value at a particular direction. iv) None of the above.	
	g)	Hessian is a..... i) A scalor number ii) A vector iii) A rectangular matrix iv) A square matrix	
	h)	Minimum can not be obtained for a quadratic function when.... i) The absolute gradient is very large ii) the absolute gradient is very small ii) The Hessian is positive definite iii) The Hessian is negative definite	
	i)	A KT point is i) Guarrenteed to be an optimum ii) A possible candidta to be an optimum iii) Only states whether the point is feasible or not. iv) None of the above	
	j)	A solution in the genetic algorithm is analogous to a) A Chromosome b) A Gene c) A Cell	

		d) A species	
	k)	Generally the cooling schedule in simulated annealing is i) Linearly increasing ii) Linearly decreasing iii) Exponentially increasing iv) Exponentially decreasing	
	l)	Which of the following method is not a biologically inspired algorithm? i) Genetic algorithm ii) Particle swarm optimization iii) Ant colony optimization iv) Simulated annealing	
1	B)	Answer any five questions	5 x 2
	a)	 In the above figure, (a,b) is the search range. x1 and x2 are two points and f(x1)>f(x2). State which of the following is correct? i) The minimum cannot lie between a and x1 ii) The minimum can not lie between x1 and x2 iii) he minimum can not lie between x2 and b iv) None of the above	
	b)	Which of the following direction is a conjugate direction when $y^{(1)}=\begin{pmatrix}2\\4\end{pmatrix}$, $H=\begin{pmatrix}2 & -2\\-2 & 8\end{pmatrix}$ Hints: $((y_i)^T H\ y_j = 0)$	

		i) $\begin{pmatrix}7\\1\end{pmatrix}$ ii) $\begin{pmatrix}1\\7\end{pmatrix}$ iii) $\begin{pmatrix}-7\\1\end{pmatrix}$ iv) $\begin{pmatrix}7\\-1\end{pmatrix}$	
	c)	The linear approximation of *f(x)* in the close proximity to a point $x^{(t)}$ is I) $f'(x-x^{(t)})$ ii) $f(x^{(t)})-\nabla f(x^{(t)})(x-x^{(t)})$ iii) $f(x^{(t)})+\nabla f(x^{(t)})(x-x^{(t)})$ iv) $\nabla f(x^{(t)})(x-x^{(t)})$	
	d)	The inversion of the $H=\begin{pmatrix}2 & 4\\-4 & 8\end{pmatrix}$ is	
	e)	In a minimization problem of $f(x_1,x_2)=(x_1-10)^2+(x_2-5)^2$, five following solutions $\begin{pmatrix}1\\2\end{pmatrix}$, $\begin{pmatrix}-2\\10\end{pmatrix}$, $\begin{pmatrix}10\\2\end{pmatrix}$, $\begin{pmatrix}7\\1\end{pmatrix}$, and $\begin{pmatrix}8\\5\end{pmatrix}$ are generated randomly. Find out two most ellite solutions.	
	f)	Which of the following terminology are associated with the partcile swarm optimization. I) Local best solution ii) Global best solution iii) Momentum iv) All of the above	
2.		Identify the following algorithm (Only name of the algorithm)	10

		Algorithm **Step 1** Choose a lower bound a and an upper bound b. Also choose a small number ϵ. Normalize the variable x by using the equation $w = (x-a)/(b-a)$. Thus, $a_w = 0$, $b_w = 1$, and $L_w = 1$. Set $k = 1$. **Step 2** Set $w_1 = a_w + (0.618)L_w$ and $w_2 = b_w - (0.618)L_w$. Compute $f(w_1)$ or $f(w_2)$, depending on whichever of the two was not evaluated earlier. Use the fundamental region-elimination rule to eliminate a region. Set new a_w and b_w. **Step 3** Is $\lvert L_w \rvert < \epsilon$ small? If no, set $k = k + 1$, go to Step 2; Else **Terminate**. Consider the following algorithm $$f(x) = x^2 + 54/x$$ a) Assume $a=0$ and $b=5$ in step 1. Find w_1, w_2, and L_w in consecutive two iterations.	
	1	b) Solve the above problem using Newton-Raphson's method for three consecutive iterations. Assume initial point $x^{(1)}=1$. Find $x^{(2)}$ and $x^{(3)}$.	9
3	a)	Find the stationary points in the following equations (i) $f(x_1, x_2) = x_1^2 + 2x_2^2 - 4x_1 - 2x_1x_2$. (ii) $f(x_1, x_2) = 10(x_2 - x_1^2)^2 + (1 - x_1)^2$.	8
	b)	What do you mean by descent direction? Find whether the given direction s at the point x is the descent for the respective functions. (i) For $f(x_1, x_2) = 2x_1^2 + x_2^2 - 2x_1x_2 + 4$, $$s = (1, 1)^T, \quad x = (2, 3)^T.$$ (ii) For $f(x_1, x_2) = x_1^4 + x_2^3 - 2x_1^2x_2^2 + 10x_1/x_2^2$, $$s = (-1, 2)^T, \quad x = (0, 1)^T.$$	8
	c)	State parallel subspace property for the optimization of a quadratic function.	4
4.	a)	State Kuhn-Tucker (KT) condition for Non-linear programming	3
	b)	Write down KT conditions for the following Maximize $3x_1^2 - 2x_2$ subject to	10

		$2x_1+x_2=4$ $x_1^2+x_2^2\le 19.4$, $x_1\ge 0$ Find out whether points $(0,4)^T$ and $(3.4,-2.8)^T$ are KT points or not.	
	c)	Minimize $f(x_1,x_2)=(x_1-2)^2+(x_2-2)^2$ Subject to $x_1+x_2\le 2$	7
5.	a)	State Frank-Wolfe algorithm for NLP. In an NLP problem, the following constraints are used: $g_1(x)=(x_1-5)^2+x_2^2-25\le 0,$ $g_2(x)=x_1+2x_2-12\le 0,$ $g_3(x)=2x_1+x_2+4\ge 0.$ Convert the above NLP program to a Linear programming problem when the initial problem $x^{(1)}=(1,1)^T$	10
	b	State the following steps with examples of Genetic algorithm	10
6	a)	State the algorithm of Particle swarm optimization. Explain local best solution and global best solution.	8
	b)	Minimize the following objective function using PSO. $f(x_1,x_2)=(x_1-5)^2+(x_2-10)^2$ Fill the following table for 2 iterations of PSO. Assume the initial velocity of all particles as $\begin{pmatrix}0\\0\end{pmatrix}$. The velocity equation is as follows $v^i(n+1)=\eta v^{(n)}+c_1.r_1(X^i(n)-X_{localbest})+c_2.r_2(X^i(n)-X_{globalbest})$ Assume $c_1=c_2=0.5$ and $r_1=r_2=0.1\wedge 0.3\,respectively$. Assume $\eta=1$. T	12

$\binom{x_1}{x_2}$	$f(x_1, x_2)$	$v(n)$	$X(n+1)$
$\binom{1}{2}$			
$\binom{5}{1}$			
$\binom{8}{10}$			
$\binom{6}{4}$			
Local best			
Global best			

	c)		
7	Write short notes (Any two)		10 x2
	a)	Simulated Annealing	
	b)	Powel's conjugate gradient algorithm	
	c)	Conjugate Gradient algorithm	
	d)	Lagranzian Duality theory	
	e)	Cauchy's (steepest descent) Method	