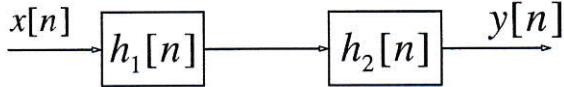## 2024

DIGITAL SIGNAL PROCESSING

Full Marks: 100
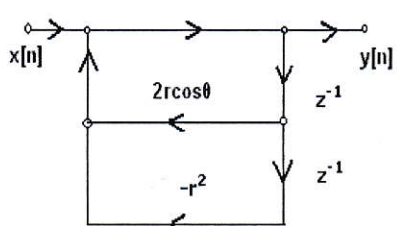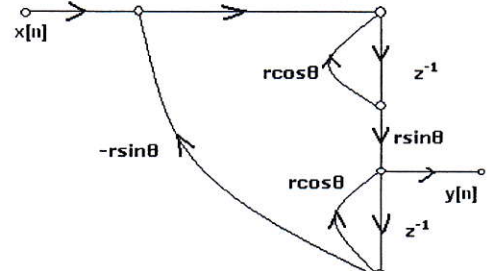
Time: Three hours

The figures in the margin indicate full marks for the questions.

*Answer **any five** questions.*

1.	a)	A causal LTI system is characterised by the difference equation $y(n)=\dfrac{1}{4}y(n-1)+\dfrac{1}{8}y(n-2)+x(n)-x(n-1)$. Find the system function '$H(z)$' and the unit impulse response '$h(n)$'.	10
	b)	Determine the inverse z-transform of $X(z)=\dfrac{z+2}{2z^2-7z+2}$ using partial fraction expansion method and if the ROC's are (i) $|z|>3$, (ii) $|z|<\dfrac{1}{2}$ and (iii) $\dfrac{1}{2}<|z|<3$.	10
2.	a)	There are two kinds of particles inside a nuclear reactor. Every second, an 'α' particle will split into eight 'β' particles and a 'β' particle will split into a 'α' particle and two 'β' particles. If there is a single 'α' particle in the reactor at time $t=0$, how many particles are there altogether at time $t=100$?	14
	b)	Two causal systems with impulse responses $h_1[n]=a\times\delta[n]+\delta[n-1]$ and $h_2[n]=b^n\times u[n]$, where $|b|<1$, are connected in cascade as shown below. Determine the frequency response $H\left(e^{j\omega}\right)$ of the overall system. Find the values of 'a' and 'b' for which $\left|H\left(e^{j\omega}\right)\right|=1$.	6
3.	a)	Consider the discrete-time system shown below. For what values of 'k' is the system BIBO stable?	10

	b)	For a low pass RC network $\left(R=1\,\text{M}\Omega \text{ and } C=1\mu F\right)$ shown below, determine the equivalent discrete-time expression for the circuit response $y(n)$, when the input is $x(t)=e^{-2t}$ and the sampling frequency is 50 Hz.	10

4.	a)	From the given pole-zero plot, determine the system transfer function and explain its filtering action.	8+4

	b)	An FIR filter (N=11) is characterized by the following transfer function: $H(z)=\sum_{n=0}^{N-1}h(n)\times z^{-n}$. Determine the magnitude response and also prove that the phase and group delays are constant.	8
5.	a)	Draw and establish the complete signal flow-graph of an 8-point DIT-FFT technique.	15
	b)	Calculate the 8-point DIT-FFT of the given sequence: $x(n)=\{1,2,3,4,4,3,2,1\}$	5

6.	a)	Show that linear phase FIR filters have symmetric impulse response characteristics, i.e., $h(n) = h(N-n-1)$.	10
	b)	Determine the system function of the two networks given below and show that they have the same poles. Network-1 Network-2	10

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