2023

DIGITAL SIGNAL PROCESSING

Full Marks: 100

Time: Three hours

The figures in the margin indicate full marks for the questions.

Answer any five questions.

| 1. | a) | Determine the output sequence of the system with impulse response | , |
|----|-----|--|-------|
| | | $h[n] = \left(\frac{1}{4}\right)^n \times u[n]$, when the input is the complex exponential sequence | 10 |
| | | $x[n] = 4e^{j\pi \frac{n}{2}}; -\infty < n < \infty.$ | |
| | b) | $x[n] = 4e^{j\pi \frac{n}{2}}; -\infty < n < \infty.$ Given that $H(z) = \frac{z+1}{z^2 - 0.9z + 0.81}$ is a causal system, find the: (a) transfer function representation (b) difference equation | 3+2+5 |
| 2. | (a) | (b) difference equation representation (c) impulse response representation. | |
| | | Show that an LTI system with system function ' $H(z)$ ' is BIBO stable if | |
| | | and only if the ROC for $H(z)$ contains the unit circle. | 5+5 |
| | b) | Discuss what will happen in z-plane if the poles in s-plane are integer | |
| | | multiples of $\left(\frac{2\pi}{T}\right)$; where "T" is the sampling time. | |
| | c) | Two causal systems with impulse responses $h_1[n] = a \times \delta[n] + \delta[n-1]$ and | |
| | | $h_2[n] = b^n \times u[n]$, where $ b < 1$, are connected in cascade as shown below. | 10 |
| | | $ \begin{array}{c} x[n] \\ \hline h_1[n] \end{array} $ $ \begin{array}{c} y[n] \\ \hline \end{array} $ | |
| | | Determine the frequency response $H(e^{j\omega})$ of the overall system. Find the | |
| | | values of 'a' and 'b' for which $\left H\left(e^{j\omega}\right)\right =1$. | |
| 3. | a) | Consider the discrete-time system shown below. For what values of 'k' is | 10 |
| | | the system BIBO stable? | |
| | | | |

| | | x(n) | |
|----|-----|---|-----|
| | | ^(1) | |
| | | | |
| | | \(\frac{\dagger{\lambda}}{\dagger}\) \(\frac{\dagger{\dagger{\lambda}}}{\dagger{\dagger{\dagger}}} \\ \frac{\dagger{\dagger{\dagger{\dagger{\dagger}}}{\dagger{\dagger{\dagger{\dagger{\dagger{\dagger}}}}} \\ \dagger{\dagger | |
| | | q(n-1) z ⁻¹ q(n) | |
| | | k/3 | |
| | | \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ | |
| | | y(n) | |
| | | + > | |
| | | | |
| | (b) | 1 of a low pass RC network $(R = 1 \text{ MIS} 2 \text{ and } C = 1 \mu F)$ shown below | |
| | | determine the equivalent discrete-time expression for the circuit response | 10 |
| | | $y(n)$, when the input is $x(t) = e^{-2t}$ and the sampling frequency is 50 Hz | 10 |
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| | | $x(t)$ $C \perp y(t)$ | |
| | | | |
| 1 | | - | |
| 4. | a) | From the given pole-zero plot, determine the system transfer function and explain its filtering action. | 8+4 |
| | | explain its intering action. | |
| | | | |
| | | | |
| | | R | |
| | | $\pi/4$ $R=0.8$ | |
| | | π_4 | |
| | | R | |
| | | तमसा सा ज्योतिर्गाय | |
| | | direction of the second | |
| | | | |
| | b) | An FIR filter (N=11) is characterized by the following transfer function: $N=1$ | 8 |
| | | $H(z) = \sum_{n=0}^{N-1} h(n) \times z^{-n}$. Determine the magnitude response and also prove | |
| | | n=0 . Becomme the magnitude response and also prove | 2 |
| 5. | a) | that the phase and group delays are constant. | |
| | , | Draw and establish the complete signal flow-graph of an 8-point DIT-FFT technique. | 12 |
| | b) | | |
| | 0) | Calculate the 8-point DIT-FFT of the given sequence: $r(n) = \begin{bmatrix} 1 & 2 & 3 & 4 & 4 & 2 & 2 \end{bmatrix}$ | 8 |
| | | $x(n) = \{1, 2, 3, 4, 4, 3, 2, 1\}$ | |
| | | | |

| 6. | a) | Discuss the design of an FIR system using window method. Why ripples are present in the passband and the stopband of the magnitude response of FIR filters? | 8+4 |
|----|----|---|-----|
| | b) | A differentiator is a continuous-time LTI system with the system function $H_C(s) = s$. A discrete-time LTI system is constructed by replacing 's' in $H_C(s)$ by the following transformation known as the bilinear transformation: $s \to \frac{2}{T_s} \left(\frac{1-z^{-1}}{1+z^{-1}} \right)$, to simulate the differentiator. Choose ' T_s ' as a part of the design procedure. i) Draw a diagram for the discrete-time system. ii) Find the frequency response of the discrete-time system. | 3+5 |

