2022

DIGITAL SIGNAL PROCESSING

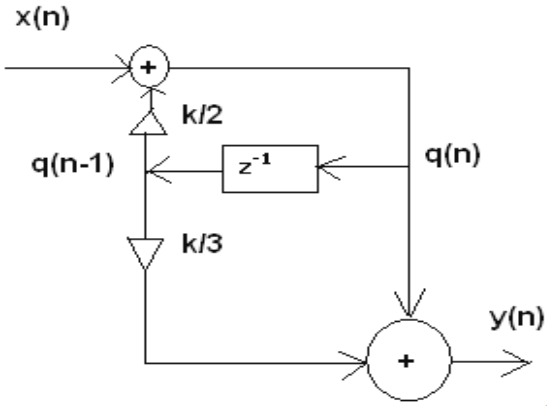
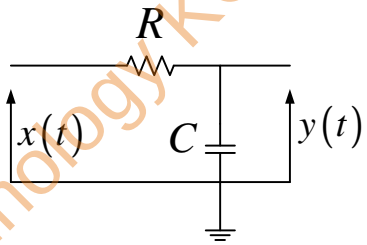
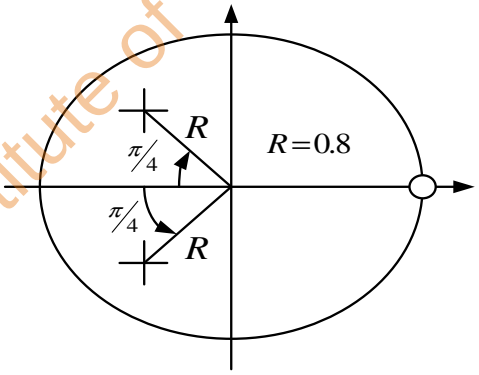
Full Marks: 100

Time: Three hours

The figures in the margin indicate full marks for the questions.

*Answer **any five** questions.*

1.	a)	Determine the output sequence of the system with impulse response $h[n]=\left(\frac{1}{4}\right)^n \times u[n]$, when the input is the complex exponential sequence $x[n]=4e^{j\pi\frac{n}{2}};\ -\infty<n<\infty$ & A = 4.	10
	b)	Given that $H(z)=\frac{z+1}{z^2-0.9z+0.81}$ is a causal system, find the: (a) transfer function representation (b) difference equation representation (c) impulse response representation.	3+2+5
2.	a) b)	Show that an LTI system with system function '$H(z)$' is BIBO stable if and only if the ROC for $H(z)$ contains the unit circle. Discuss what will happen in z-plane if the poles in s-plane are integer multiples of $\left(\frac{2\pi}{T}\right)$; where 'T' is the sampling time.	5+5
	c)	Two causal systems with impulse responses $h_1[n]=a\times\delta[n]+\delta[n-1]$ and $h_2[n]=b^n\times u[n]$, where $\lvert b\rvert<1$, are connected in cascade as shown below. $x[n] \rightarrow \boxed{h_1[n]} \rightarrow \boxed{h_2[n]} \rightarrow y[n]$ Determine the frequency response $H(e^{j\omega})$ of the overall system. Find the values of 'a' and 'b' for which $\lvert H(e^{j\omega})\rvert = 1$.	10
3.	a)	Consider the discrete-time system shown below. For what values of 'k' is the system BIBO stable?	10

			
	b)	For a low pass RC network $(R=1\ \text{M}\Omega$ and $\text{C}=1\mu F)$ shown below, determine the equivalent discrete-time expression for the circuit response $y(n)$, when the input is $x(t)=e^{-2t}$ and the sampling frequency is 50 Hz. 	10
4.	a)	From the given pole-zero plot, determine the system transfer function and explain its filtering action. 	8+4
	b)	An FIR filter (N=11) is characterized by the following transfer function: $H(z)=\sum_{n=0}^{N-1}h(n)\times z^{-n}$. Determine the magnitude response and also prove that the phase and group delays are constant.	8
5.	a)	Draw and establish the complete signal flow-graph of an 8-point DIT-FFT technique.	12
	b)	Calculate the 8-point DIT-FFT of the given sequence: $x(n)=\{1,2,3,4,4,3,2,1\}$	8

6.	a)	Show that linear phase FIR filters have symmetric impulse response characteristics, i.e., $h(n) = h(N-n-1)$.	10
	b)	A differentiator is a continuous-time LTI system with the system function $H_C(s) = s$. A discrete-time LTI system is constructed by replacing 's' in $H_C(s)$ by the following transformation known as the bilinear transformation: $s \rightarrow \frac{2}{T_S}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$, to simulate the differentiator. Choose 'T_S' as a part of the design procedure. i) Draw a diagram for the discrete-time system. ii) Find the frequency response of the discrete-time system.	5+5

XXXXXXXXXXXXX