

**2024**  
**SIGNALS AND SYSTEMS**

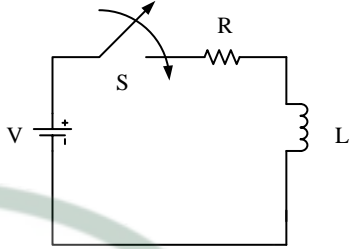
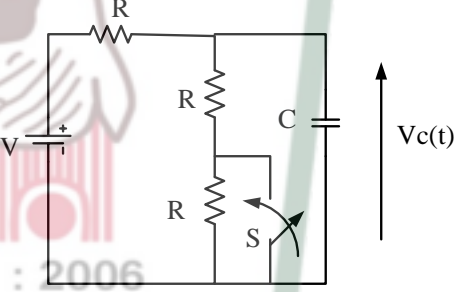
Full Marks: 100

Time: Three hours

*The figures in the margin indicate full marks for the questions.*

Answer **any five** questions.

1.	<p>a) Show that (i) <math>x_e(t) = \frac{[x(t) + x(-t)]}{2}</math> and (ii) <math>x_o(t) = \frac{[x(t) - x(-t)]}{2}</math> ; where '<math>x_e(t)</math>' and '<math>x_o(t)</math>' are the even and the odd components of the signal '<math>x(t)</math>'.</p>	5+5 = <b>10</b>
	<p>b) If the input applied to an LTI system with impulse response '<math>h(t)</math>' is '<math>x(t)</math>', deduce the output '<math>y(t)</math>' from the LTI system.</p>	<b>10</b>
2.	<p>a) Show that the complex exponential '<math>e^{j\omega_0 t}</math>' is periodic with time period '<math>T_0 = k \times \left(\frac{2\pi}{\omega_0}\right)</math>'.</p>	<b>5</b>
	<p>b) Prove that the power of the energy signal is zero and the energy of the power signal is infinite.</p>	<b>5</b>
	<p>c)</p> <div style="text-align: center;"> </div> <p>The above system consists of one continuous time integrator, a scalar multiplier '<math>a</math>' and a summer. Write a differential equation that relates the instantaneous output '<math>y(t)</math>' with the input '<math>x(t)</math>'.</p>	<b>10</b>
3.	<p>a) In the given figure, find the continuous-time Fourier series (CTFS) representation of '<math>S(t)</math>':</p> <div style="text-align: center;"> </div> <p>Hence, show that only the odd harmonics of the series exist.</p>	8+2 = <b>10</b>

	b)	Find the Fourier transform of a rectangular pulse $\Pi\left(\frac{t}{\tau}\right)$ ; where ' $\tau$ ' is the pulse width of the pulse. Hence find the zero crossings of the spectrum of the rectangular pulse.	6+4 = <b>10</b>
4.	a)	<p>In the RL circuit shown below, a unit step input voltage '<math>V \times u(t)</math>' is applied with all initial conditions assumed to be zero.</p>  <p>Using Laplace transform, find the current through the circuit. Find the voltage across the resistance and the inductance. Hence plot the voltage across the resistance and the inductance.</p>	5+2+3 = <b>10</b>
	b)	<p>The circuit given below is initially in the steady state with the switch 'S' open. The switch is closed at <math>t = 0</math>.</p> <p>i) Find <math>V_C(t)</math>.</p> <p>ii) Determine the final value of '<math>V_C(t)</math>' and verify it from the final value of Laplace transform.</p> 	7+3 = <b>10</b>
5.	a)	What are band-limited signals? State the sampling theorem for a low-pass band-limited signal. Hence show that the spectrum of sampled waveform, in case of impulse sampling, is the repetition of the spectrum of low-pass band-limited signal.	1+2+10 = <b>13</b>
	b)	Show that the transfer function of a Zero-Order Hold (ZOH) circuit is given by $\frac{\tilde{X}_a(s)}{X(s)} = \frac{1 - e^{-sT_s}}{s}$ ; where the symbols have their usual meaning.	<b>7</b>
6.	Write short notes on <b>any two</b> from the following:		10+10 = <b>20</b>
	a)	BIBO stability of LTI system.	
	b)	DC component of a signal using the polar form representation of the Fourier series.	
	c)	Fourier transform of unit step function ( $u(t)$ ).	
	d)	Reconstruction with a low-pass filter: Sinc interpolation.	

XXXXXXXX