Total number of printed pages:4

UG/3^{rd}/UECE303

2021

SIGNALS AND SYSTEMS

Full Marks: 100

Time: Three hours

The figures in the margin indicate full marks for the questions.

Answer any five questions.

1. a) Write the representation of color video signal in mathematical form and list out the dependant and independent variables in this case. 4

   b) Explain the steps involved in converting an analog signal into an M-ary digital signal? 4

   c) Distinguish between conjugate symmetric and conjugate anti-symmetric signals. Show that any complex valued signal could be expressed as a sum of these two types. 4

   d) What is the necessary condition on ω for the signal $x[n] = \sin \omega n$ to become periodic? Give one example each for periodic and non-periodic $x[n]$. 4

   e) Given $x(t) = \begin{cases} 2; & 0 \le t \le 2 \\ 4 - t; & 2 \le t \le 4 \\ 0; & \text{otherwise} \end{cases}$. Evaluate $x(-2t + 3)$ by sequentially performing the time shifting, time-scaling and time-inversion operations. Plot the signal at each stages of transformation. 4

2. a) Define unit impulse function $\delta(t)$. Show that $\delta(at) = \frac{1}{a}\delta(t)$. 4

   b) Check the linearity and time-invariance of the 2+2 following systems.

(i) $y[n] = nx[n]$ (ii) $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$

c) Check whether the systems shown below are causal, stable and memoryless. 4

(i) $y(t) = x(2-t)$ (ii) $y[n] = \sum_{k=-\infty}^{n} x[k+2]$

d) Show that the zero-state response of a linear time-invariant (LTI) system is given by the convolution of input signal and impulse response. 4

e) Given $x(n) = \{1,2,\underset{\uparrow}{3},4\}$and $h(n) = \{\underset{\uparrow}{1},1,1\}$. Find $x[n] * h[n]$. 4

3. a) Discuss how one can define the notion of inner product in a Complex Vector Space (CVS) of power signals? List the properties satisfied by this definition. 4

b) Given a set of orthogonal functions $\{g_1(t), g_2(t), \ldots\ldots, g_N(t)\}$, show that the best value of scalar multiples $c_i$to approximate any given signal $f(t)$ as $f_a(t) = \sum_{i=1}^{N} c_i g_i(t)$ is given by $c_i = \langle f(t), g_i(t)\rangle / E_i$ where $E_i = \langle g_i(t), g_i(t)\rangle$. 8

c) The one cycle of the output of a half wave rectifier is given as $f(t) = \begin{cases} V_m \sin \omega t \,; 0 \le t < \frac{T}{2} \\ 0 \quad ; \quad \frac{T}{2} \le t < T \end{cases}$. Find the approximate representation of this periodic waveform using trigonometric Fourier series. Here $\omega T = 2\pi$. 8

4. a) If $x(t) \overset{LT_u}{\longleftrightarrow} X(s)$ are Laplace transform pairs, then find the unilateral LT of the following: 2+2+3+3

(i) $x(t-a)$

(ii) $e^{-at}x(t)$

(iii) $\frac{dx(t)}{dt}$

(iv) $\int_{-\infty}^{t} x(\tau)d\tau$

b) Find the unilateral Laplace transform of the following functions. 3+2

(i) $\cos \omega_0 t\, u(t)$ (ii) $e^{-at} \cos \omega_0 t\, u(t)$

c) Find the zero-state response of the system represented by the following differential equation $(D^2 + 5D + 6)y(t) = -(4 + 3D)u(t)$, where $u(t)$ is the unit step function. 5

5. a) Evaluate the Fourier Transform and compare the spectrum of the following functions- (i) $\delta(t)$ (ii) $\frac{1}{2\pi}$ 4

b) If $x_1(t) \overset{FT}{\leftrightarrow} X_1(\omega)$ and $x_2(t) \overset{FT}{\leftrightarrow} X_2(\omega)$ are Fourier transform (FT) pairs, find the FT of the following: (i) $x_1(t) * x_2(t)$ (ii) $x_1(t)x_2(t)$ 3+3

c) For an energy signal, show that auto-correlation function and energy spectral density are FT pairs. 5

d) Determine the Fourier Transform of the impulse train signal $\delta_T(\mathrm{t}) = \sum_{n=-\infty}^{+\infty} \delta(t - nT)$ 5

6. a) State and prove sampling theorem for band limited signals. 5

b) Describe the frequency response of an ideal low pass filter. Discuss why it isn't practically feasible. 5

c) Show that the dimension of Fourier series orthogonal set for the representation of a discrete-time periodic 5 signal is finite. Explain this with the help of an example where the period N=6.

d) Starting with the analysis and synthesis expressions of discrete-time Fourier series (DTFS) derive the corresponding expressions for discrete-time Fourier transform (DTFT). 5

7. a) Explain with the help of an example why it is essential to specify region of convergence (ROC) for a bilateral Z transform? 4

b) Exploring the connection between DTFT and Z-transform, find the formula for evaluating the inverse Z-transform. 5

c) Determine the Z transform of the signal x[n] = n u[n]. 5

d) Determine the inverse Z transform of 6

$$X(z)=\frac{z^2}{0.5-1.5z+z^2} \quad \text{for ROC } |z|<0.5$$