

Total No. of printed pages = 4

19/3rd Sem/UECE303

2021

**SIGNALS AND SYSTEMS**

Full Marks – 100

Time – Three hours

The figures in the margin indicate full marks  
for the questions.

Answer any *five* questions.

1. (a) Describe how a digital color image is represented. 4
- (b) Find the energy and power in a signal  $x(t) = e^{-at+j\omega t}u(t)$  if  $0 < \alpha < \infty$ . 4
- (c) Distinguish between digital signals and discrete-time signals. 4

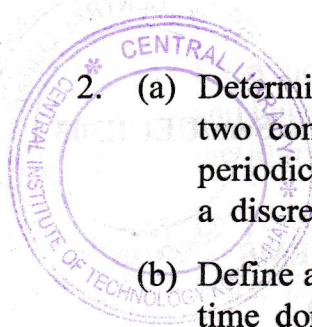
- (d) Show how the signal  $x(t) \begin{cases} A; 2 \leq t \leq 4 \\ At/2; 0 \leq t \leq 2 \\ 0; \text{Otherwise} \end{cases}$

could be transformed to  $x(1-2t)$  as a sequence of time-inversion, time-scaling and time-shifting operation. Find the even and odd part of  $x(t)$  and plot them as function of time.

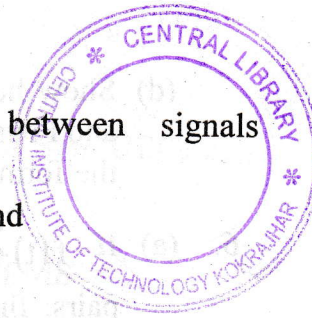
4+4=8

[Turn over





2. (a) Determine the condition under which sum of two continuous-time periodic signal is also periodic. Describe the necessary condition for a discrete-time signal to be periodic. 6
- (b) Define a unit impulse function in continuous-time domain. List and prove any of its two properties. 6
- (c) Show that how a discrete-time signals can be expressed as a shifted and scaled version of unit-impulse signal. 4
- (d) Describe a triangular pulse in terms of unit-step function. 4
3. (a) Draw the waveforms having half-wave symmetry :
- (i) With an additional odd symmetry
- (ii) With an additional even symmetry
- (iii) With no additional symmetry.
- (b) Find the best coefficients of linear expansion of a given signal  $f(t)$  in terms of an orthogonal set  $\{g_1(t), g_2(t), g_3(t) \dots g_N(t)\}$  of functions defined over the same interval. 8
- (c) Show how one can derive exponential Fourier series from the trigonometric Fourier series. 6



4. (a) Find the convolution between signals

$$x_1(t) = \begin{cases} 1; 0 \leq t < 1 \\ 0; \text{Elsewhere} \end{cases} \quad \text{and}$$

$$x_2(t) = \begin{cases} -1; -1 \leq t < 0 \\ 1; 0 \leq t < 1 \\ 0; \text{Elsewhere} \end{cases} \quad \text{and plot the result as}$$

a function of time. 10

(b) Derive the condition for stability of a linear system in terms of its impulse response. 4

(c) Check for the linearity, time-invariance and causality of the following systems : 6

$$(i) y(t) = \frac{dx(t)}{dt} \quad (ii) y(t) = 2x(t) + x(2t).$$

5. (a) Find the unilateral Laplace transform of

$$\frac{d^2x(t)}{dt^2}. \quad 4$$

(b) Evaluate the total response of the system represented by the differential equation  $(D^2 + 5D + 6)y(t) = (D + 1)x(t)$ . Here  $x(t) = u(t)$ ,  $y(0^-) = 2$  and  $y'(0^-) = -1$ . 8

(c) State and prove initial value theorem. 4

- (d) Show how the relationship between Laplace and Fourier transform could be utilized to find the formula for inverse Laplace transform. 4
6. (a) If  $f(t) \xleftrightarrow{\text{FT}} F(\omega)$  are Fourier transform (FT) pairs, find the FT of the following :  
 2+2+2=6
- (i)  $tf(t)$
- (ii)  $f(at)$
- (iii)  $f(t-T)$ .
- (b) Describe the frequency response of the ideal low pass filter. Using Paley-Wiener criterion, explain why this is not practically realizable. 6
- (c) Find the Fourier transform of  $x(t) = e^{-a|t|}$  and plot its magnitude and phase spectrum for various positive values of  $a$ . 8
7. (a) State and prove Sampling theorem for band-limited signals. 6
- (b) Find the DTFT of the unit-step function. 8
- (c) Evaluate the inverse Z-transform of  $\frac{z}{(z-1)^2}$  if the signal is causal. 6

