Total No. of printed pages = 4

19/3rd Sem/UECE303

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2021

SIGNALS AND SYSTEMS

Full Marks – 100

Time – Three hours

The figures in the margin indicate full marks for the questions.

Answer any *five* questions.

1. (a) Describe how a digital color image is represented. 4

   (b) Find the energy and power in a signal $x(t) = e^{-at+j\omega t}u(t)$ if $0 < \alpha < \infty$. 4

   (c) Distinguish between digital signals and discrete-time signals. 4

   (d) Show how the signal $x(t)\begin{cases} A; 2 \le t \le 4 \\ At/2; 0 \le t \le 2 \\ 0; \text{Otherwise} \end{cases}$ could be transformed to $x(1-2t)$ as a sequence of time-inversion, time-scaling and time-shifting operation. Find the even and odd part of x(t) and plot them as function of time. 4+4=8

[Turn over

2. (a) Determine the condition under which sum of two continuous-time periodic signal is also periodic. Describe the necessary condition for a discrete-time signal to be periodic. 6

(b) Define a unit impulse function in continuous-time domain. List and prove any of its two properties. 6

(c) Show that how a discrete-time signals can be expressed as a shifted and scaled version of unit-impulse signal. 4

(d) Describe a triangular pulse in terms of unit-step function. 4

3. (a) Draw the waveforms having half-wave symmetry : 6

(i) With an additional odd symmetry

(ii) With an additional even symmetry

(iii) With no additional symmetry.

(b) Find the best coefficients of linear expansion of a given signal f(t) in terms of an orthogonal set $\{g_1(t), g_2(t), g_3(t)\ldots g_N(t)\}$ of functions defined over the same interval. 8

(c) Show how one can derive exponential Fourier series from the trigonometric Fourier series. 6

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4. (a) Find the convolution between signals

$$x_1(t) = \begin{cases} 1; 0 \le t < 1 \\ 0; \text{Elsewhere} \end{cases}$$ and

$$x_2(t) = \begin{cases} -1; -1 \le t < 0 \\ 1; 0 \le t < 1 \\ 0; \text{Elsewhere} \end{cases}$$ and plot the result as

a function of time. 10

(b) Derive the condition for stability of a linear system in terms of its impulse response. 4

(c) Check for the linearity, time-invariance and causality of the following systems : 6

(i) $y(t) = \dfrac{dx(t)}{dt}$ (ii) $y(t) = 2x(t) + x(2t)$.

5. (a) Find the unilateral Laplace transform of $\dfrac{d^2x(t)}{dt^2}$. 4

(b) Evaluate the total response of the system represented by the differential equation $(D^2 + 5D + 6)y(t) = (D + 1)x(t)$. Here $x(t) = u(t)$, $y(0^-) = 2$ and $y'(0^-) = -1$. 8

(c) State and prove initial value theorem. 4

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(d) Show how the relationship between Laplace and Fourier transform could be utilized to find the formula for inverse Laplace transform. 4

6. (a) If $f(t) \overset{FT}{\leftrightarrow} F(\omega)$ are Fourier transform (FT) pairs, find the FT of the following : 2+2+2=6

(i) $tf(t)$

(ii) $f(at)$

(iii) $f(t-T)$.

(b) Describe the frequency response of the ideal low pass filter. Using Paley-Wiener criterion, explain why this is not practically realizable. 6

(c) Find the Fourier transform of $x(t)=e^{-a|t|}$ and plot its magnitude and phase spectrum for various positive values of *a*. 8

7. (a) State and prove Sampling theorem for band-limited signals. 6

(b) Find the DTFT of the unit-step function. 8

(c) Evaluate the inverse Z-transform of $\frac{z}{(z-1)^2}$ if the signal is causal. 6

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