Total number of printed pages: 3 **UG/7th Semester/UCSE701**

2024

Advance Algorithms (Back)

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

Answer ***any five*** *questions.*

1.	a)	(i) Solve the recurrence relation. 5×2=10 $T(n)=2T(\sqrt{n})+1$, $T(1)=1$. *(ii)* Set up a recurrence relation for the following algorithm and solve it. **ALGORITHM** *S(n)* //Input: A positive integer *n* **Step 1. if** $n = 1$ Step 2. **return** 1 **Step 3. else** **Step 4. return** S$(n-1) + 1/n$	10
	b)	Prove that the average case time-complexity of quick sort is O(n log n)	10
2.	a)	Suppose we're doing a sequence of n operations (numbered 1, 2, 3,...) on a data structure in which the ith operations cost is as follows: $cost = \begin{cases} 1 & if\ i \neq power\ of\ 2 \\ i & if\ i = power\ of\ 2 \end{cases}$ For example, the following table shows the costs for each of the first few operations: operation number: 1 2 3 4 5 6 7 8 9 ... cost: 1 2 1 4 1 1 1 8 1 ... Use aggregate analysis to determine amortized cost per operation.	10
	b)	Find the amortized cost per operation of augmented stack using potential analysis	10
3.	a)	Solve the following 0/1 knapsack problem by Dynamic programming (weight limit W=11):	10

Item	Weight(w_i)	Value(v_i)
1	1	1
2	2	6

3	5	18
4	6	22
5	7	28

<table>
<tr><td></td><td>b)</td><td>Suppose the letters a, b, c, d, e, f have probabilities ½, 1/4, 1/8, 1/16, 1/32, 1/32 respectively. Find the Huffman code for letter a, b, c, d, e, f.</td><td>10</td></tr>
<tr><td>4.</td><td>a)</td><td>Compute the time complexity of Ford-Fulkerson algorithm to find the maximum flow of the graph.</td><td>5</td></tr>
<tr><td></td><td>b)</td><td>State the Max-flow min-cut theorem for network flow analysis. Trace the execution of Ford-Fulkerson algorithm to find the maximum flow of the graph given in figure 1.

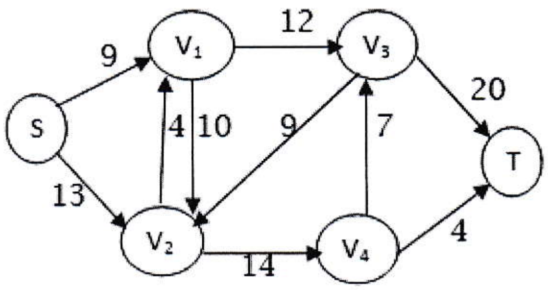

Figure 1</td><td>3+12</td></tr>
<tr><td>5.</td><td></td><td>Solve the APSP problem using Floyd-Warshall's algorithm for the graph given in figure 2:

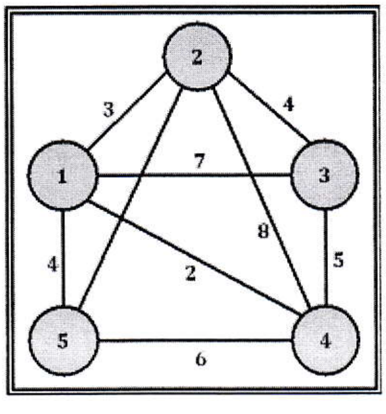

Figure 2</td><td>20</td></tr>
<tr><td>6</td><td>a)</td><td>Define the classes P and NP. Discuss diagrammatically the relations among P class, NP class, NP hard and NP complete.</td><td>10</td></tr>
<tr><td></td><td>b)</td><td>Trace the execution of vertex cover problem using Approximation algorithm of the graph given in figure 3. Also find the Approximation ratio (p(n)).</td><td>10</td></tr>
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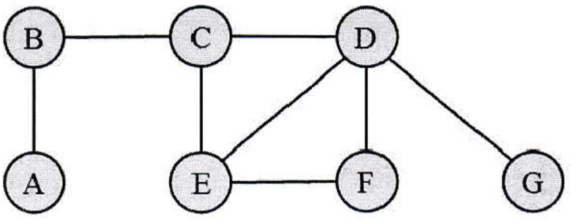
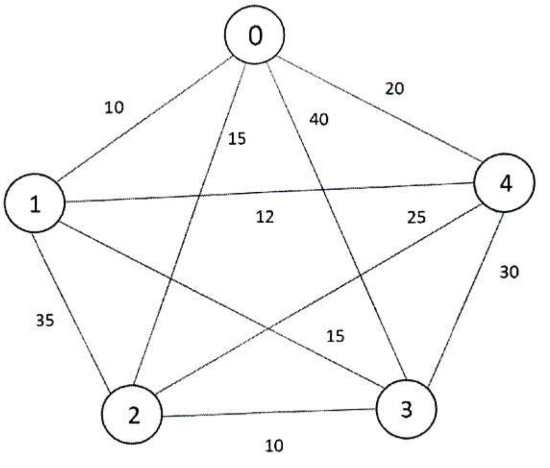
		 Figure 3	
7.	a)	Apply the 2-approximate and 1.5-approximate algorithm for the Travelling Salesman Problem of the graph given in figure 4.  Figure 4	10+10=20

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