CENTRAL INSTITUTE OF TECHNOLOGY KOKRAJHAR
(Deemed to be University)
KOKRAJHAR :: BTR :: ASSAM :: 783370

END – SEMESTER EXAMINATION
UG

Session: July-December, 2023 Semester: 7th Time: 3Hrs. Full Marks: 100

Course Code: UCSE701 Course Title: Advance Algorithms

ANSWER ANY 5 QUESTIONS

1. (a) Prove that if $f(n) = a_m n^n + a_{m-1} n^{n-1} + \cdots\cdots + a_1 n + a_0$ then $f(n) = \theta(n^m)$
   (b) Find the asymptotic upper bound for T(n) of the following recurrence. Assume that T(1) = 1.
   $T(n) = T(n-1) + \frac{1}{n}$
   (c) Using master's theorem find the time complexity for the following recurrences using master's theorem:
   (i) $T(n)=4T(n/2)+n^2\sqrt{n}$
   (ii) $T(n) = 16T\left(\frac{n}{4}\right)+n^2$

2. Find the minimum number of operations required for the following matrix chain multiplication using dynamic programming:
   $$A(10\times20)*B(20\times50)*C(50\times1)*D(1\times100)$$

3. (a) Suppose we're doing a sequence of n operations (numbered 1, 2, 3,...) on a data structure in which the ith operations cost is as follows:
   $$cost = \begin{cases} 1 \; if \; i \neq power \; of \; 2 \\ i \;\; if \; i = power \; of \; 2 \end{cases}$$
   For example, the following table shows the costs for each of the first few operations:

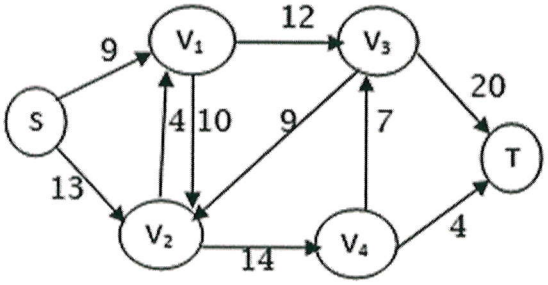
operation number:	1	2	3	4	5	6	7	8	9 ...
cost:	1	2	1	4	1	1	1	8	1 ...

   Find the amortized cost per operation of augmented stack using potential analysis

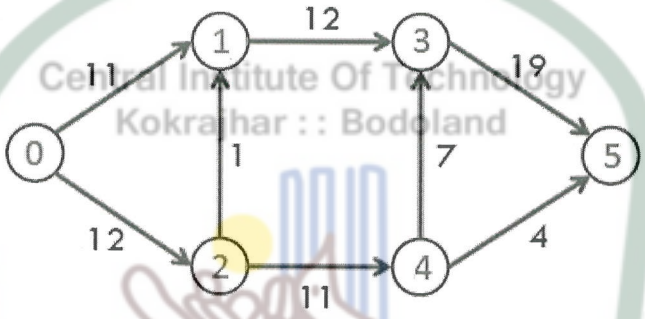
   (b) Find the amortized cost per operation using aggregate analysis of the Table-Insert operation(Algorithm-1) given below:

```
Algorithm - 1: Table-Insert (T,x)
1   if T.size == 0
2       allocate T.table with 1 slot
3       T.size = 1
4   if T.num == T.size
5       allocate newTable with 2*T.size slots
6       insert all items in T.table into newTable
7       free T.table
8       T.table = newTable
9       T.size = 2*T.size
10  insert x into T.table
11  T.num = T.num + 1
```

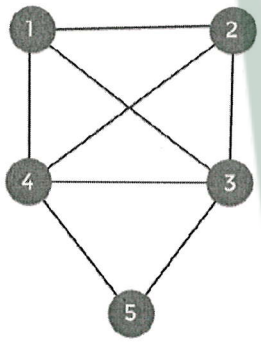
4. (a) Ford-Fulkerson algorithm to find the maximum flow of the graph. Find the augmenting path of the following graph.



(b) Trace the execution of Ford-Fulkerson algorithm to find the maximum flow of the following graph.



5. Illustrate the steps to solve the travelling salesman problem of the following graph using Branch & Bound



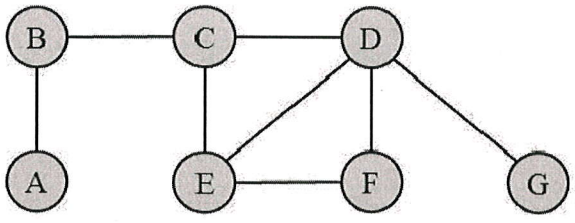
(a) Graph G

	1	2	3	4	5
1	∞	20	30	10	11
2	15	∞	30	10	11
3	3	5	∞	2	4
4	19	6	18	∞	3
5	16	4	7	16	∞

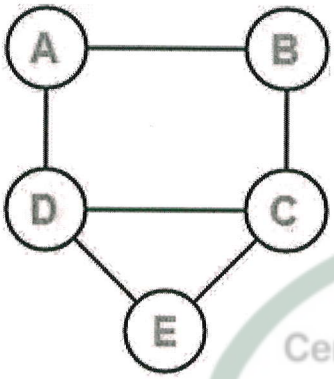
(b) The adjacency matrix of G

6. (a) Define the classes P and NP. Discuss diagrammatically the relations among P class, NP class, NP hard and NP complete.
(b) Assume that 3-SAT problem as NP-Complete problem. Reduce Clique decision problem to 3-SAT problem.

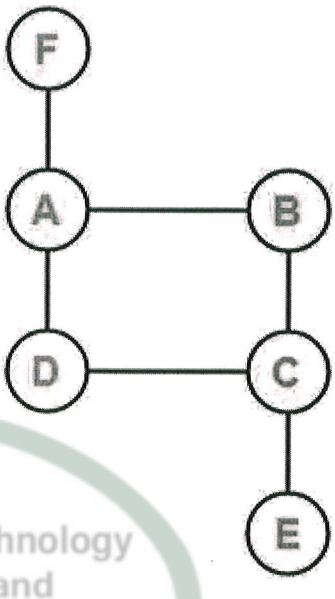
7. (a) Apply the 2-approximate on the following graph for vertex cover. Also find the Approximation ratio (p(n)).



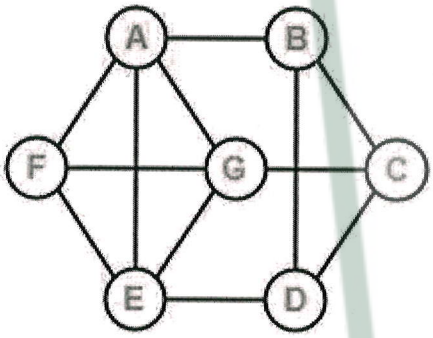
(b) How does the concept of Hamiltonian circuits relate to the Traveling Salesman Problem? Determine whether following graphs are Hamiltonian graph or not. Provide a proper justification for your answer in terms of Hamiltonian cycle/circuits. [2+8]



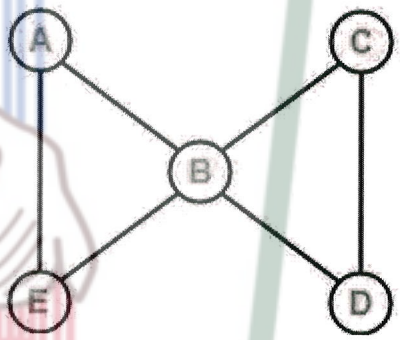
(a)



(b)



(c)



(d)