Total number of printed pages: 3   **UG(B.Tech)/4th Semester/UCSE403**

2024

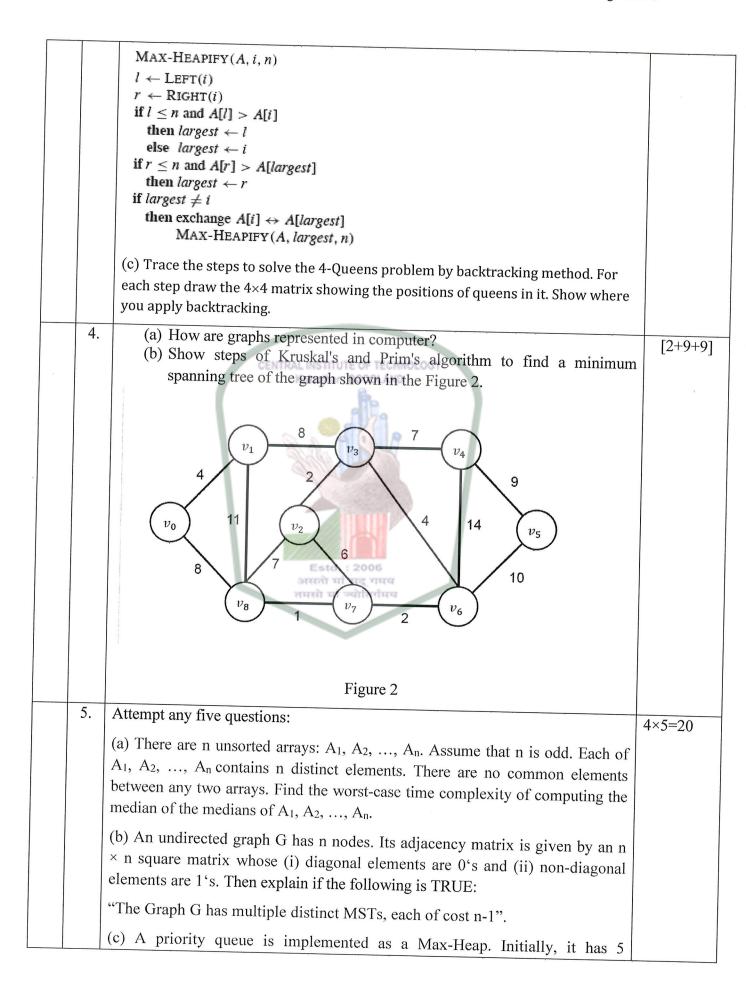### Design and Analysis of Algorithm

Full Marks: 100

Time: Three hours

The figures in the margin indicate full marks for the questions.

		Attempt all five 20×5=100	

	1.	(a) Define and differentiate asymptotic notations (O, Θ, Ω).
		
		(b) Solve the following recurrence relation:
		
		    $T(n)=2T(n^{1/2}) + \log(n)$, $T(1) = 1$.
		(c) Find the time complexity of the following algorithm
		
		int func (mt n)
		{
		  if (n == 1)
		    return (1);
		  else
		    return (func (n-1) + func (n-1));
		}
			[6+7+7]

	2.	(a) Illustrate the operation of Partition in the context of the quicksort algorithm on the array: $A = (13,19,9,5,12,8,7,4,21,2,6,11)$ Assuming that the last element (that is, 11) is chosen as the pivot element, show the steps involved in one pass of the partitioning process that places the pivot element in its final position in the sorted list. (b) Write the algorithm to merge two sorted list.	[12+8]
	3.	(a) Define a Heap data structure and its properties (Min Heap and Max Heap). (b) Illustrate the operation of Max-Heap(A,3) on the array $A=<27,17,3,16,13,10,1,5,7,12,4,8,9,0>$.	[2+10+8]

MAX-HEAPIFY(A, i, n)
$l \leftarrow$ LEFT(i)
$r \leftarrow$ RIGHT(i)
if $l \leq n$ **and** $A[l] > A[i]$
  then $largest \leftarrow l$
  else $largest \leftarrow i$
if $r \leq n$ **and** $A[r] > A[largest]$
  then $largest \leftarrow r$
if $largest \neq i$
  then exchange $A[i] \leftrightarrow A[largest]$
    MAX-HEAPIFY$(A, largest, n)$

(c) Trace the steps to solve the 4-Queens problem by backtracking method. For each step draw the 4×4 matrix showing the positions of queens in it. Show where you apply backtracking.

4.	(a) How are graphs represented in computer? (b) Show steps of Kruskal's and Prim's algorithm to find a minimum spanning tree of the graph shown in the Figure 2.		[2+9+9]

Figure 2

5.	Attempt any five questions:		4×5=20

(a) There are n unsorted arrays: $A_1, A_2, \ldots, A_n$. Assume that n is odd. Each of $A_1, A_2, \ldots, A_n$ contains n distinct elements. There are no common elements between any two arrays. Find the worst-case time complexity of computing the median of the medians of $A_1, A_2, \ldots, A_n$.

(b) An undirected graph G has n nodes. Its adjacency matrix is given by an n × n square matrix whose (i) diagonal elements are 0's and (ii) non-diagonal elements are 1's. Then explain if the following is TRUE:

"The Graph G has multiple distinct MSTs, each of cost n-1".

(c) A priority queue is implemented as a Max-Heap. Initially, it has 5

elements. The level-order traversal of the heap is: 10, 8, 5, 3, 2. Two new elements 1 and 7 are inserted into the heap in that order. Find the level-order traversal of the heap after the insertion of the elements 1 and &.

(d) If one uses a straight two-way merge sort algorithm to sort the following elements in ascending order 20, 47, 15, 8, 9, 4, 40, 30, 12, 17, then find the order of these elements after the second pass (2nd merge) of the algorithm.

(e) Let P be a QuickSort Program to sort numbers in ascending order using the first element as pivot. Let t_1 and t_2 be the number of comparisons made by P for the inputs {1, 2, 3, 4, 5} and {4, 1, 5, 3, 2}, respectively. Determine which is larger, t_1 or t_2 with proper justification?

(f) Let X be a problem that belongs to the class NP. Then explain if the following is TRUE with diagram:
"If X is NP-hard, then it is NP-complete "