CENTRAL INSTITUTE OF TECHNOLOGY KOKRAJHAR
(Deemed to be University)
KOKRAJHAR :: BTR :: ASSAM :: 783370

END – SEMESTER EXAMINATION
UG

Session: **Janu-June 2025** Semester: **IV** Time: **3Hrs**. Full Marks: **100**
Course Code: **UCSE402** Title: **Probability Theory & Random Process**

Answer Question 1 and any *four* of the remaining questions

1. (a) The probability of an impossible event is...?
   (i) 0 (ii) 1 (iii) any value in between 0 and 1 (iv) None of these

(b) Two unbiased coins are tossed, then P(getting at least one head) is
   (i) $\frac{1}{4}$ (ii) $\frac{2}{4}$ (iii) $\frac{3}{4}$ (iv) $\frac{4}{4}$

(c) Given P(A) = $\frac{5}{13}$, P(B) = $\frac{7}{13}$ and P(A ∩ B) = $\frac{3}{13}$, then P(A|B) is
   (i) $\frac{2}{7}$ (ii) $\frac{3}{4}$ (iii) $\frac{3}{7}$ (iv) $\frac{1}{4}$

(d) Given Var(X) = 1, var(Y) = 9 and Cov(X, Y) = 1, then ρ_{XY} is
   (i) $\frac{1}{3}$ (ii) 0 (iii) -1 (iv) -$\frac{1}{3}$

(e) Calculate $P(0 < X < 0.5; 0.5 < Y < 1)$ from the given p.d.f in (1)

$$f(x, y) = \begin{cases} 1 & if\ 0 < x < 1, 0 < y < 1 \\ 0 & otherwise \end{cases} \tag{1}$$

(f) Events A and B are independent implies that
   (i) P(AB) = P(A)P(B) (ii) P(A|B) = P(A) (iii) P(B|A) = P(B) (iv) All the above

(g) Find the value of $\boldsymbol{k}$ from the given probability distribution.

X	1	2	4	5	8	Otherwise
P(X)	0.1	0.2	0.2	k	0.2	0

(h) Ten data points are generated from the equation $y = 2x + 1$. The correlation coefficient value of these data points is
   (i) 1 (ii) 0 (iii) 0.5 (iv) None of the above

(i) Classification problem is well suited in
   (i) Bayes' theorem (ii) Markov chain (iii) Linear regression (iv) Any of the above

(j) Find the eigen values of $\begin{bmatrix} 1 & 8 \\ 2 & 1 \end{bmatrix}$

2 x 10

2. (a) A real number X is selected uniformly at random in the interval [0, 10] then find the value of P($2 \leq X \leq 7$) and P($2 \leq X|X \leq 7$).
(b) Define Poisson distribution. Find the mean and variance of the Poisson distribution.

5 + 5 + 10

3. (a) State and prove Bayes' theorem.
(b) Company A produces defective 10%, B produces defective products 20%, and C produces defective products 5%. A product is chosen randomly and found to be defective. What is the probability that it is from company A. [Assume that choosing a company is an equally likely event].

10 + 10

4. Convert the matrix A to its diagonalized form. Hence find the value of A^5. What are the criteria for a matrix to be diagonalizable?

$$A = \begin{bmatrix} 6 & -1 \\ 2 & 3 \end{bmatrix}$$

12 + 5 + 3

5. Assume there are two telephone operators in Kokrajhar city: Airtel and Jio. Each year, 40% of Airtel customers switch to Jio, while 30% of Jio customers switch to Airtel. Also, assume that their initial market shares were 70% for Airtel and 30% for Jio. Draw the transition diagram and write the transition matrix. What will be their market shares after two years? What will be their market shares in the long run? Define markov properties.

3 + 6 + 7 + 4

6. Answer any four:
(a) State and prove Chebyshev Inequality.
(b) Prove that $-1 \leq \rho \leq 1$ [ρ implies Correlation coefficient].
(c) Write a short note on moment generating function.
(d) Given $y = mx + c$, calculate Cov(X, Y).
(e) If events A and B are independent, prove that $\bar{A}$ and $\bar{B}$ are also independent.
(f) From the given joint distribution, find the value of P(X = Y).

P(X, Y)	Y = 0	Y = 1	Y = 2
X = 0	0.10	0.04	0.02
X = 1	0.08	0.20	0.06
X = 2	0.06	0.14	0.30

4 x 5

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