Total number of printed pages = 9

19/4th Sem/UCSE 403

2022

DESIGN AND ANALYSIS OF ALGORITHM

Full Marks – 100

Time – Three hours

The figures in the margin indicate full marks for the questions.

GROUP – A

Multiple choice questions (answer any *ten*) : 1×10=10

1. Consider the following three claims :

   (i) $(n+k)^m = O(n^m)$ where k and m are constants

   (ii) $2^{n+1} = O(2^n)$

   (iii) $2^{2n+1} = O(2^n)$

   Which of the following claims are correct ?

   (a) (i) and (ii) (b) (i) and (iii)

   (c) (ii) and (iii) (d) (i), (ii) and (iii)

[Turn over

2. What is the correct definition of an algorithm ?

   (a) An algorithm is a step by step instructions to solve a problem

   (b) An algorithm is a process of baking bread

   (c) An algorithm is a software used to compute numbers

   (d) An algorithm is the process of breaking problems

3. Which of the following is not a type of graph in computer science ?

   (a) Undirected graph

   (b) Bar graph

   (c) Directed graph

   (d) Weighted graph

4. Kruskal's algorithm is a _____.

   (a) Divide and conquer algorithm

   (b) Greedy algorithm

   (c) Dynamic Programming

   (d) Approximation algorithm

5. Which of the following is false about Prim's algorithm ?

   (a) It is a greedy algorithm

   (b) It constructs MST by selecting edges in increasing order of their weights

   (c) It never accepts cycles in the MST

   (d) It can be implemented using the Fibonacci heap

6. What is the time complexity of Prim's algorithm ?

   (a) $O(|E| \log |E|)$

   (b) $O(E \log V)$

   (c) $O(E^2)$

   (d) $O(V \log E)$

7. In what manner is a state-space tree for a backtracking algorithm constructed ?

   (a) Depth-first search

   (b) Breadth-first search

   (c) Twice around the tree

   (d) Nearest neighbour first

8. Which of the problems cannot be solved by backtracking method ?

   (a) n-queen problem

   (b) Graph coloring problem

   (c) Hamiltonian circuit problem

   (d) Travelling Salesman problem

9. Which of the following problems should be solved using dynamic programming ?

   (a) Mergesort

   (b) Binary search

   (c) Longest common subsequence

   (d) Quicksort

10. What is a randomized Quicksort ?

   (a) The leftmost element is chosen as the pivot

   (b) The rightmost element is chosen as the pivot

   (c) Any element in the array is chosen as the pivot

   (d) A random number is generated which is used as the pivot

11. In what position does the array for heap sort contains data ?

(a) 0

(b) 1

(c) –1

(d) Anywhere in the array

12. Assuming P ! = NP, which of the following is true ?

(a) NP-complete = NP

(b) NP-complete $\cap$ P = NULL

(c) NP-hard = NP

(d) P = NP-complete.

GROUP – B

Answer any *five* of the following questions : 18×5=90

1. (a) Define and differentiate asymptotic notations : (O, θ, $\cap$).

(b) Draw the recursive tree for the recurrence relation :

$$T(n) = T(n/3)+T(2n/3)+n$$

(c) Solve the recurrence relation : 6+6+6=18

$T(n) = 2T(n/2)+1,\ T(1) = 1.$

2. (a) What does dynamic programming have in common with divide-and-conquer ? What is a principal difference between them ?

(b) Design an efficient algorithm for computing the binomial coefficient C(n, k) that uses no multiplications. What are the time and space efficiencies of your algorithm ?

(c) Solve the all-pairs shortest-path problem using DP for the digraph given in figure 1. 3+5+10=18

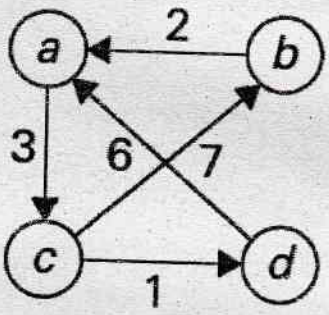


Figure 1

3. (a) Illustrate the operation of Partition in the context of the quicksort algorithm on the array :

$$A = (2, 8, 7, 1, 3, 5, 6, 4)$$

Assuming that the last element (that is, 4) is chosen as the pivot element, show the steps involved in one pass of the partitioning process that places the pivot element in its final position in the sorted list.

(b) Write the algorithm of Quicksort. 10+8=18

4. (a) Illustrate the operation of build Max Heap (A,8) on the array A = <4,1,3,2,16,9,10,14,8,7> to make a max heap tree.

(b) Prove that the average case time-complexity of merge sort is O(n log n). 9+9=18

5. Show steps of Kruskal's and Prim's algorithm to find a minimum spanning tree of the graph shown in the Figure 2. 9+9=18

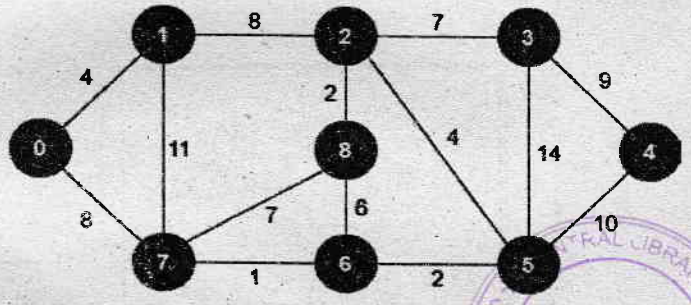


Figure 2

6. (a) Trace the steps to solve the 4-Queens problem by backtracking method. For each step draw the 4×4 matrix showing the positions of queens in it. Show where you apply backtracking.

(b) Apply backtracking technique to solve the 3-coloring problem for the following graph (figure 3). Also generate the state space tree. 9+9=18

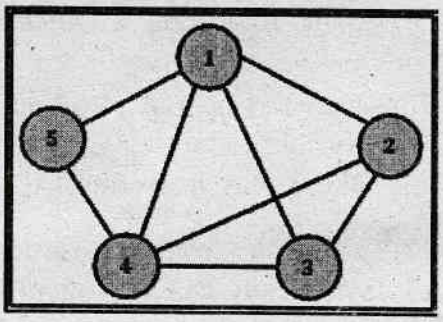


Figure 3

7. (a) Find out Hamiltonian cycle of the graph given in figure 4 and also draw the state space tree.

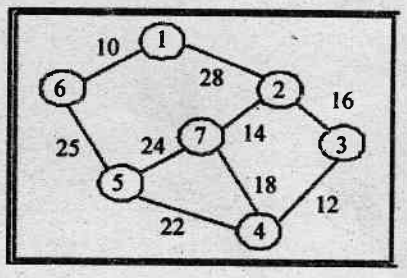
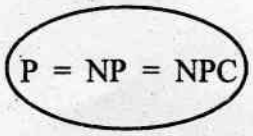


Figure 4

(b) Which of the following diagrams do not contradict the current state of our knowledge about the complexity classes P, NP, and NPC (NP-complete problems) ? 8+10=18

(i)



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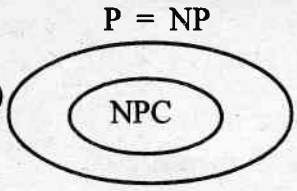

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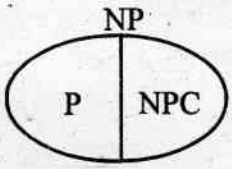

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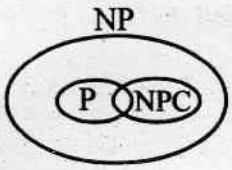

(v)