Total number of printed pages = 4

19/4th Sem/UCSE 402

2022

PROBABILITY AND RANDOM PROCESS

Full Marks – 100

Time – Three hours

The figures in the margin indicate full marks for the questions.

Answer any *five* questions.

1. (a) Prove that $P(A+B) \leq P(A)+P(B)$

   (b) Consider the joint probability distribution in the given table :

X \ Y	0	1	2
0	1/9	2/9	1/9
1	2/9	2/9	0
2	1/9	0	0

Are X and Y being independent ? Explain.Find the value of $P(x = 0 \mid y = 2)$. Find the value of E(XY). 5+15=20

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2. (a) Define Moment generating function.

(b) Find the Moment generating function of Binomial distribution.

(c) Find the Mean and Variance of Binomial distribution using the above result.

5+5+10=20

3. (a) State and prove the Bayes' theorem. 10

(b) Assume that during the pandemic situation there were 300 students, 60 faculties and 80 office staff were in the CIT campus. There was a COVID-19 test program and it was found that 40 students, 6 faculties and 12 office staff were positive. Among the positive cases only one had to admit to the hospital. What is the probability that the admitted person was a faculty person ? 10

4. (a) Assume in Kokrajhar there are two telephone operators, Airtel and Vodafone. Initially, the Airtel customers were 80% and rest were Vodafone customers. But due to some lucrative offers launched by Vodafone, every month 30% of customers switches from

Airtel to Vodafone. On the other hand, due to some connection issue, every month 20% of customers switches from Vodafone to Airtel.

Describe the above problem in the transition system and probability matrix representation. What is the % of customers of Airtel after two months. When it became stable position then what will be % of Airtel and Vodafone ? 15

(b) Describe the Markov properties. 5

5. (a) Prove that the value of Correlation coefficient is within [–1, 1].

(b) Estimate the regression line from the given data set :

X	1	2	3	4
Y	3	4	8	9

(c) Find the Eigen value and Eigen vector of the following matrix : 6+7+7=20

$$\begin{pmatrix} 1 & 2 \\ 3 & -4 \end{pmatrix}$$

6. Answer the following questions : 5×4=20

(a) State and prove the Cauchy-Schwartz's inequality.

(b) Define Spectral decomposition of a matrix.

(c) Define the Covariance and interpretation.

(d) State the Central limit theorem.