

Total number of printed pages = 4

19/4th Sem/UCSE 402

2022

PROBABILITY AND RANDOM PROCESS

Full Marks – 100

Time – Three hours

The figures in the margin indicate full marks for the questions.

Answer any *five* questions.

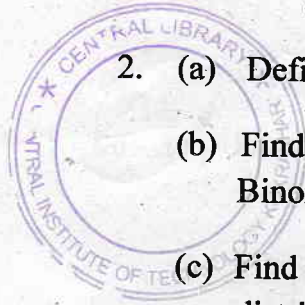
1. (a) Prove that $P(A+B) \leq P(A)+P(B)$
- (b) Consider the joint probability distribution in the given table :

X \ Y	0	1	2
0	1/9	2/9	1/9
1	2/9	2/9	0
2	1/9	0	0

Are X and Y being independent ? Explain. Find the value of $P(x = 0 | y = 2)$. Find the value of $E(XY)$.

5+15=20

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2. (a) Define Moment generating function.
- (b) Find the Moment generating function of Binomial distribution.
- (c) Find the Mean and Variance of Binomial distribution using the above result.

$$5+5+10=20$$

3. (a) State and prove the Bayes' theorem. 10
- (b) Assume that during the pandemic situation there were 300 students, 60 faculties and 80 office staff were in the CIT campus. There was a COVID-19 test program and it was found that 40 students, 6 faculties and 12 office staff were positive. Among the positive cases only one had to admit to the hospital. What is the probability that the admitted person was a faculty person? 10
4. (a) Assume in Kokrajhar there are two telephone operators, Airtel and Vodafone. Initially, the Airtel customers were 80% and rest were Vodafone customers. But due to some lucrative offers launched by Vodafone, every month 30% of customers switches from

Airtel to Vodafone. On the other hand, due to some connection issue, every month 20% of customers switches from Vodafone to Airtel.

Describe the above problem in the transition system and probability matrix representation. What is the % of customers of Airtel after two months. When it became stable position then what will be % of Airtel and Vodafone ? 15

(b) Describe the Markov properties. 5

5. (a) Prove that the value of Correlation coefficient is within $[-1, 1]$.

(b) Estimate the regression line from the given data set :

X	1	2	3	4
Y	3	4	8	9

(c) Find the Eigen value and Eigen vector of the following matrix : 6+7+7=20

$$\begin{pmatrix} 1 & 2 \\ 3 & -4 \end{pmatrix}$$

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6. Answer the following questions : $5 \times 4 = 20$

- (a) State and prove the Cauchy-Schwartz's inequality.
- (b) Define Spectral decomposition of a matrix.
- (c) Define the Covariance and interpretation.
- (d) State the Central limit theorem.

