MATHEMATICS – II

Full Marks – 70

Pass Marks – 21

Time – Three hours

The figures in the margin indicate full marks for the questions.

GROUP – A

1. (a) A function f(x) is defined as follows : 3

$f(x) = 2x^2 + 3, \quad 0 \leq x < 2$

$= 3x - 5, \quad 2 < x \leq 4$

find f(0), f(1), f(3).

(b) Find the Domain of the function : 2

$$f(Z) = \frac{3z^3 + 4z - 1}{(z-2)(z-1)}$$

2. Find the limit (any *two*) : 2×3=6

(i) $\lim_{x \to 0} \log(1+x)$

(ii) $\lim_{x \to \infty} \frac{2x(x+1)}{(x+4)(x-1)}$

(iii) $\lim_{x \to 0} \frac{8^x - 2^x}{x}$

(iv) $\lim_{x \to 2} \frac{x^2 - 5x + 6}{x^2 - 7x + 10}$

3. Find $\frac{dy}{dx}$ of $y = a^x$ using first principle. 3

4. Find $\frac{dy}{dx}$ (any *two*) : 2×3=6

(i) $y = e^{x \sin x}$

(ii) $y = (\sec x)^{\tan x}$

(iii) $x = a \cos^3 0,\ y = b \sin^3 0$

5. (a) Find the points on the curve $y = x^3 - 2x^2 + x$ at which the tangents are parallel to X-axis. Also find the tangents at these points. 4

(b) If $y = (\tan^{-1}x)^2$ show that $(1+x^2)^2 y_2 + 2x(1+x^2) y_1 - 2 = 0$. 3

(c) Show that $f(x) = x^3 - 6x^2 + 24x + 4$ has neither maxima nor minima. 3

GROUP – B

6. Integrate any *three* of the following : $2\times3=6$

(i) $\int \frac{dx}{a + bx^2}$

(ii) $\int \sin mx \sin nx \, dx$

(iii) $\int \frac{e^{5x} + e^{3x}}{e^x + e^{-x}} dx$

(iv) $\int \sqrt{1+\sin 2x} \, dx$

7. Integrate any *three* of the following : $3\times3=9$

(i) $\int_0^1 \frac{dx}{\sqrt{x+1} - \sqrt{x}}$

(ii) $\int_0^{\pi/2} \sin^2 x \cos^2 x \, dx$

(iii) $\int_0^{\pi/2} \frac{dx}{2 + \cos x}$

(iv) $\int x^3 \cos x \, dx$

(v) $\int \frac{x - 13}{x^2 - 5x + 6} dx$

8. (a) Find the circumference of the circle $x^2 + y^2 = a^2$ 2

(b) Evaluate : $\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx$ 4

Or

$$\lim_{x \to \infty} \left[\frac{1}{n} + \frac{n^2}{(n+1)^3} + \frac{n^2}{(n+2)^3} + \ldots\ldots + \frac{1}{8n} \right]$$

GROUP – C

9. Answer any *five* questions : 5×2=10

(i) Write the locus of a point equidistant from a fixed point (h, k), where the distance is 'k'.

(ii) Change $r = a \sin 2\theta$ to Cartesian form.

(iii) Show that the points $(0, 0)$, $(0, \pi/3)$ and $(0, 2\pi/3)$ are the vertices of an equilateral triangle.

(iv) In what ratio the line segment joining A(3, 4) and B(5, –7) will be divided by Y-axis ?

(v) Find the area of the quadrilateral formed by the points (0, 3), (2, –1), (5, 0) and (4, 6).

(vi) Show that the lines $2x - y + 8 = 0$, $3x + y+2 = 0$ and $4x + 3y - 4 = 0$ are concurrent.

(vii) Find the eccentricity, focii, length of the L. R and the equation of the directrix of the hyperbola $16x^2 - 9y^2 = 144$.

10. Answer any *three* questions : $3\times3=9$

(i) Find the equation of the straight line which passes through the point (2, 3) and whose sum of the intercepts on X-axis and Y-axis equals 10.

(ii) Find the equation of the tangent to the circle $x^2 + y^2 - 6x - 3y - 2 = 0$ at (2, –2).

(iii) Express $x + \sqrt{3y} = 8$ in the perpendicular form. Also find 'α'.

(iv) Find the equation of the parabola with focus at (1, –3) and directrix $x - 2y + 3 = 0$.

(v) The latus rectum of an ellipse is half of its major axis. Find the value of the two eccentricities.