MATHEMATICS – III

Full Marks – 70

Pass Marks – 28

Time – Three hours

The figures in the margin indicate full marks
for the questions.

GROUP – A

(Differential Equation)

1. (a) What are the order and degree of the
   following differential equation ?   1+1=2

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = \left(\frac{d^2y}{dx^2}\right)^2$$

   (b) Form the differential equation of which
   solution is $y = A \cos 6x + B \sin 6x$.   2

   (c) Solve :   2

$$\left(1+x^2\right)\frac{dy}{dx} = 1+y^2$$

[Turn over

2. Solve any *three* : $3 \times 4 = 12$

   (a) $x\dfrac{dy}{dx} + \cot y = 0$, given $y = \dfrac{\pi}{4}$, $x = \sqrt{2}$

   (b) $x^2 dy + y (x + y) dx = 0$

   (c) $(x+1)\dfrac{dy}{dx} - y = e^x (x+1)^2$

   (d) $\dfrac{dy}{dx} - \dfrac{\tan y}{1+x} = (1+x) e^x \sec y$

   (e) $x\dfrac{dy}{dx} - 3y = x^2$

3. Answer any *two* : $2 \times 5 = 10$

   (a) In a circuit an inductance of 2 henries and a resistance of 20 ohms are connected in series with an emf E volts. If the current is zero when $t = 0$, find the current at the end of ·01 sec if $E = 10$ volts assuming it obeys voltage law

   $$L\dfrac{dI}{dt} + RI = E \; ; \text{ I being current}$$
   $$L \text{ inductance}$$

(b) Solve :

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 6e^{3x} + 7e^{-2x}$$

(c) Solve :

$$D^2(D^2+4)y = 96x^2$$

(d) Solve :

$$(e^y + 1)\cos x\ dx + e^y\ \sin x\ dy = 0$$

GROUP – B

(Graphics)

Answer any *three*.

4. The number of revolutions x (per minute) and power y (h.p) of a diesel engine is given in the table :

x :	400	500	600	700	750
y :	580	1030	1420	1880	2100

Draw the straight line of best fit and find its equation. 6

5. The law of a machine is $P = aW + b$ where P is the effort and W the load in kg. Sketch the graph showing the relation between P and W, given

P :	60	70	90	100	125
W :	225	270	380	430	550

   Find P when W = 500.   6

6. In the following table some observed value of x and y are given :

x :	2	3	4	5	6	7
y :	4	5	5·71	6·25	6·67	7

   Fit the law $xy = ax + by$ to this data.   6

7. A simply supported beam carries a concentrated load P at its mid point ; corresponding to various values of P, the maximum deflection y is measured

P :	100	120	140	160	180	200
y :	·45	·55	·6	·7	·8	·85

   Fit a law of the form $y = a + bP$.   6

8. Find a root of the following equation :6

   $$x^3 - 6x - 13 = 0$$

GROUP – C

(Statistics)

9. Find mean, median and mode from the following table : $\qquad$ 2+3+3=8

Class	Frequency
0–10	7
10–20	8
20–30	20
30–40	10
40–50	5

10. Find standard deviation from the following data : $\qquad$ 4

Class	Less than cumulative frequency
0–4	4
4–8	12
8–12	14
12–16	15

Or

Calculate correlation coefficient between X and Y : $\qquad$ 4

X :	78	36	98	25	75	82	90
Y :	84	51	91	60	68	62	86

11. An urn contains 13 balls numbering 1 to 13. Find the probability that a ball selected at random is a ball with number that is a multiple of 3 or 4.

3

<p style="text-align:center">Or</p>

An anti-aircraft gun can take a maximum of four shots on enemy's plane moving from it. The probabilities of hitting the plane at first, second, third and fourth shots are ·4, ·3, ·2 and ·1 respectively. Find the probability that the gun hits the plane.

3

GROUP – D

(Vectors and Solid Geometry)

12. Answer any *three* : $3×3=9$

(a) Find the ratio in which the line joining the points (4, 4, −10) and (−2, 2, 4) is divided by yz-plane.

(b) If α, β, γ are the angles which a line makes with the axes, prove that

$$\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2$$

(c) Show that (0, 7, 10), (−1, 6, 6) and (−4, 9, 6) form an isosceles right angled triangle.

(d) If the position vectors of P and Q are $2i + 3j - 7k$ and $4i - 3j + 4k$ respectively, find $\overrightarrow{PQ}$ and determine its direction cosines.

(e) Forces of magnitudes 5 and 3 units acting in the directions $6i + 2j + 3k$ and $3i - 2j + 6k$ respectively, act on a particle which is displaced from the point $(2, 2, -1)$ to $(4, 3, 1)$. Find the work done by the forces.