Total No. of printed pages = 5

MATHEMATICS–II

Full Marks – 70

Pass Marks – 21

Time – Three hours

The figures in the margin indicate full marks for the questions.

GROUP – A

1. (a) If $f(x) = \tan x$,

   prove that $f(x+y) = \dfrac{f(x)+f(y)}{1-f(x)f(y)}$ 3

   (b) Find the domain of the function

   $f(x) = \dfrac{1}{\sqrt{x^2-4}}$ 3

2. Evaluate any *two* :

   (a) $\displaystyle \lim_{x \to 0} \frac{\sqrt{1+2x} - \sqrt{1-2x}}{x}$

[Turn over

(b) $\displaystyle\lim_{\theta \to 0} \frac{1-\cos\theta}{\theta^2}$

(c) $\displaystyle\lim_{x \to \infty} \frac{2x^2+8x+3}{5x^2+7x-1}$

3. Find $\dfrac{dy}{dx}$ of any *three* if $3\times2=6$

(i) $y = \log(ax^2 + bx + c)$

(ii) $y = x^2 \log x$

(iii) $y = \cos^{-1}(4x^3 - 3x)$

(iv) $x^4 + x^2y^2 + y^4 = 0$

(v) $x = a\,\sec^2\theta, \ y = a\,\tan^2\theta$

4. Find $\dfrac{dy}{dx}$ (any *two*) : $3\times2=6$

(a) $y^x - x^y$

(b) $x = a\cos^3t, \ y = a\sin^3t$

(c) $y = \tan^{-1}\dfrac{\cos x}{1+\sin x}$

(d) $y = (\cot x)^{\sin x}$

5. Find the equation of the tangent to the curve $y^2 = 4x + 5$ which is parallel to the line $y = 2x + 1$.

3

6. Find the extremum values of the function $f(x) = 2x^3 - 21x^2 + 36x - 20$.

3

GROUP–B

7. Integrate (any *three*) :

$3 \times 3 = 9$

(i) $\int (x+3)\sqrt{x^2 + 6x + 5} \ dx$

(ii) $\int \frac{1}{x} \log (\log x) \ dx$

(iii) $\int \sin^5 x \ \cos x \ dx$

(iv) $\int \sin 2x \ \cos 6x \ dx$

(v) $\int \cos^2 x \ dx$

8. Evaluate any *two* :

$3\frac{1}{2} \times 2 = 7$

(i) $\int_0^1 \frac{x \ dx}{\sqrt{1+x^2}}$

(ii) $\displaystyle\int_{0}^{\pi/2} \sin^3 x \cos x \, dx$

(iii) $\displaystyle\int_{0}^{\pi/4} \sec^4 x \, dx$

(iv) $\displaystyle\int_{0}^{1} \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx$

9. Find the area bounded by the curves $y^2 = 4x$ and $x^2 = 4y$. 　　　　　　　　　　　　　　　4

GROUP – C

10. The points $(11, 2)$, $(3, -14)$, $(-7, -9)$ are the three vertices of a rectangle. Find the co-ordinates of the fourth vertex. 　　　　　　　　　　3

11. Find the equation of the line which is parallel to the line $5x + 4y + 7 = 0$ and passes through the point $(2, -3)$. 　　　　　　　　　　3

12. Find the equation of the circle passing through the points $(1, 3)$, $(1, -2)$ and $(-1, 1)$. 　　4

13. Find the co-ordinates of the foci, length of the latus rectum and lengths of the major and minor axes of the ellipse $3x^2 + 4y^2 = 48$. 4

14. Find the equation of the parabola whose focus is (3, 0) and directrix is n = –3. 4

15. (i) Find the slope of the line joining the points (1, 5) and (–4, 2). 1

(ii) Find the centre and radius of the circle $x^2 + y^2 = 36$. 1