Total No. of printed pages = 6

**Sc-202/Maths-II/2nd Sem/2015/M**

# MATHEMATICS – II

Full Marks – 70

Pass Marks – 21

Time – Three hours

The figures in the margin indicate full marks for the questions.

## GROUP – A

1. Answer any *five* questions : $5 \times 1 = 5$

   (a) If the distance between (2, x) and (1, 9) is $5\sqrt{2}$, find x.

   (b) Write down the equation of a line parallel to x-axis at a distance of 5 units from x-axis.

   (c) Reduce the equation $3x + 4y - 12 = 0$ to intercept form.

[Turn over

(d) Write down the condition of tangency of the line $y = mx + c$ to the circle $x^2 + y^2 = a^2$.

(e) Write down the latus rectum of the parabola $y^2 = -8x$.

(f) Find the axes of the hyperbola $16x^2 - 9y^2 = 144$.

2. Answer any *three* questions :

(a) (i) Find the condition that the point (a, b) should lie on the line joining the points (2, 3) and (– 4, –1). 3

(ii) Find the co-ordinate of a point which divides the line joining the points (7, 8) and (–6, 11) internally in the ratio 5:7. 2

(b) (i) If the sum of the square of the distance of a variable point from the two points (–1, 2) and (3, 4) are constant, find the locus of the point. 3

(ii) Write down the slopes of the line parallel and perpendicular to the line $3x + 2y - 3 = 0$. 2

(c) (i) Find the equation to the line passing through (3, 5) and perpendicular to the line joining (4, 2) and (2, 8). 3

(ii) Find the centre and radius of the circle $x^2 + y^2 - 6x + 4y - 12 = 0$. 2

(d) Find the equation of the circle passing through the points (1, 1), (2, –1), (3, 2). 5

(e) (i) Find the focus, length of latus rectum, equation of directrix of the parabola $y^2 = px$ which passes through (1, 2). 3

(ii) Find the length of latus rectum, equation of directrices of the ellipse $12x^2 + 9y^2 = 144$. 2

## GROUP – B

3. (a) Find the domain of the function 2

$$f(x)=\frac{1}{x(x-2)},\ x\in \mathbb{R}$$

(b) If $f(x)=\frac{x-1}{|x-1|}$, find the values of f(–1), f(2). 1

(c) If $f(x) = \log x$, show that $f(xy) = f(x) + f(y)$. 1

(d) Examine the continuity of the function defined as follows at $x = 1$. 2

$$f(x) = x + 2, \quad -1 \leq x < 1$$
$$= 4 - x, \quad 1 \leq x < 2$$

4. Evaluate the following limits. (any *two*) $2\times2=4$

(i) $\lim\limits_{x\to3} \dfrac{x^4 - 81}{x-3}$

(ii) $\lim\limits_{x\to0} \dfrac{x}{\sqrt{1+x} - \sqrt{1-x}}$

(iii) $\lim\limits_{\theta\to0} \dfrac{\tan\theta}{\theta}$

5. Find $\dfrac{dy}{dx}$ (any *three*) : $3\times2=6$

(i) $y = xe^{\tan x}$

(ii) $y = \dfrac{\sin x}{1 + \cos x}$

(iii) $x^3 + y^3 = a^3$

(iv) $y = \sin^{-1}\sqrt{1-x^2}$

(v) $y = \log (\sin 2x)$

6. Find the equation of the tangent and normal to the curve $y = (x - 1)(x - 2)$ at the point where the curve cuts x-axis. 4

7. Find the maximum and minimum values of the function $y = 4x^3 - 3x^2 - 6x + 1$. 4

8. If $y = e^{ax} \cos bx$, show that 4

$$\frac{d^2y}{dx^2} - 2a\frac{dy}{dx} + \left(a^2 + b^2\right)y = 0.$$

## GROUP – C

9. Integrate the following. (any *four*) $2\frac{1}{2} \times 4 = 10$

(a) $\int \tan^2 x \, dx$

(b) $\int \sin^5 x \cos^2 x \, dx$

(c) $\int xe^x \, dx$

(d) $\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}dx$

(e) $\int \frac{dx}{x^2-6x+5}$

(f) $\int \frac{dx}{\left(x^2-2x+3\right)^{1/2}}$

10. Evaluate. (any *one*) 4

(a) $\int_0^{\pi/2} \sin^5 x\,dx$

(b) $\int_0^{\pi/2} \log \sin x\,dx$

11. Evaluate the limit (using Integration). 4

$$\lim_{n\to\infty}\left[\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+.....+\frac{1}{2n}\right]$$

12. Find the area of the portion enclosed by the parabolas $y^2 = x$ and $x^2 = y$. 4