Total No. of printed pages = 11

END SEMESTER EXAMINATION – 2021

Semester : 1st (New)

Subject Code : Sc-102

MATHEMATICS – I

Full Marks : 70

Time – Three Hours

The figures in the margin indicate full marks
for the questions.

Instruction :

All questions of PART – A and PART – B are compulsory.

PART – A

Marks – 25

1. Choose the correct answer of the following
   questions : $1 \times 10 = 10$

   (i) The value of i^{109} is

   (a) $-i$ (b) i

   (c) -1 (d) 1

(ii) The modulus of $a - ib$ is

(a) $a + ib$ (b) $\sqrt{a - ib}$

(c) $\sqrt{a^2 + b^2}$ (d) $\sqrt{a^2 - b^2}$

(iii) If a polynomial $p(x)$ has no factor common to the factors of $x^3 - 1$, then the partial fraction of the proper fraction $\dfrac{p(x)}{x^3 - 1}$ be of the form

(a) $\dfrac{Ax^2 + Bx + C}{x^3 - 1}$

(b) $\dfrac{C}{x - 1} + \dfrac{A + Bx}{x^2 + x + 1}$

(c) $\dfrac{A}{x - 1} + \dfrac{Bx^2 + Cx + D}{x^2 + x + 1}$

(d) $\dfrac{A}{x - 1} + \dfrac{B}{(x - 1)^2} + \dfrac{C}{(x - 1)^3}$

(iv) If $^{n+1}C_3 = 2 \times {}^n C_2$, then the value of n is

(a) 3   (b) 4

(c) 5   (d) 6

(v) How many arrangements can be made out of the letters of the word 'POLYTECHNIC' ?

(a) $11!$

(b) $\dfrac{11!}{2!}$

(c) $11!-2!$

(d) $^{11}p_2$

(vi) Number of terms in $(2-x^2)^{15}$ is

(a) 16

(b) 15

(c) 14

(d) 30

(vii) Coefficient of middle term in expansion of

$\left(x-\dfrac{x^3}{5}\right)^8$ is

(a) $\dfrac{14}{625}$

(b) $\dfrac{70}{625}$

(c) $\dfrac{14}{125}$

(d) $\dfrac{70}{125}$

(viii) The value of $\log_{81} 27$ is

(a) $\dfrac{1}{4}$

(b) $\dfrac{1}{2}$

(c) $\dfrac{5}{4}$

(d) $\dfrac{3}{4}$

(ix) If A is a matrix of order m×n and B is a matrix of order n×p, then the order of AB is

(a) p×m   (b) p×n

(c) n×p   (d) m×p

(x) If A is a skew symmetric matrix of order 3×3, then the sum of the diagonal elements of A is

(a) 0   (b) 6

(c) 9   (d) 3

2. Choose the correct statement :   1×5=5

(i)   (a) $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B$

     (b) $\sin(A+B)\sin(A-B) = \cos^2 A - \sin^2 B$

     (c) $\sin(A+B)\sin(A-B) = \sin^2 A - \cos^2 B$

(ii)   (a) $\cos 2A = \dfrac{1-\tan^2 A}{1+\tan^2 A}$

(b) $\sin 2A = \dfrac{2\tan A}{1-\tan^2 A}$

(c) $\tan 2A = \dfrac{2\tan A}{1+\tan^2 A}$

(iii) (a) $\sin 3\theta = 4\sin^3\theta - 3\sin\theta$

(b) $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

(c) $\sin 3\theta = 3\sin^3\theta - 4\sin\theta$

(iv) In any $\triangle ABC$

(a) $a^2 = b^2 + c^2 - 2bc\cos B$

(b) $a^2 = b^2 + c^2 + 2bc\cos A$

(c) $a = b\cos C + c\cos B$

(v) (a) $\sin^{-1}(x) + \sin^{-1}(-x) = \dfrac{\pi}{2}, x \in [-1,1]$

(b) $\cos^{-1}(x) + \cos^{-1}(-x) = \pi, x \in [-1,1]$

(c) $\tan^{-1}(x) + \cot^{-1}(-x) = \dfrac{\pi}{2}, x \in \mathbb{R}$

3. Fill in the blanks : 1×5=5

(i) The volume of a sphere of radius 6 unit is _____.

(ii) The base radius of a cone is 7 units. If the height of the cone is 24 units, its lateral surface is _____.

(iii) In the Simpson's Rule : $\text{Area} = \dfrac{S}{3}[A + 2D + 4E]$, the letter 'S' stands for _____.

(iv) If m_1 and m_2 be the slopes of two perpendicular lines, then the relation between m_1 and m_2 is _____.

(v) The equation of a straight line passing through $(0, -1)$ and parallel to x-axis is _____.

4. Write true or false : 1×5=5

(i) If two columns of a determinant are same (identical), then the value of the determinant will be zero.

(ii) The value of $\sin(-1230°)$ is $\dfrac{1}{2}$.

(iii) The principal value of $\cos^{-1}(\cos\dfrac{5\pi}{4})$ is $\dfrac{\pi}{4}$.

(iv) In a triangle ABC, if the sides are a=7,b=5 and c=8, then the value of A will be 60^0.

(v) If a base of field 55m and number of ordinates are 11, then the breadth of each strip is equal to 5m.

PART – B

Marks – 45

5. Answer any *five* of the following questions :

$2\times5=10$

(i) If $x=1-i$, then prove that $x^2-2x+2=0$.

(ii) Find the argument of $2+2\sqrt{-3}$.

(iii) In how many different ways can the letters of the word PANDEMIC be arranged without changing the order of the vowels in the word ?

(iv) Write the general term in the expansion of

$\left(9x-\dfrac{1}{3\sqrt{x}}\right)^{18}$ and find its 13th term.

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(v) Prove that

$$\log 2 + 16 \log \frac{16}{15} + 12 \log \frac{25}{24} + 7 \log \frac{81}{80} = 1.$$

(vi) Insert three G.M.s between 1 and $\frac{1}{16}$.

(vii) If $A = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B^T = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$, then find $(A+2B)^T$.

6. Resolve into partial fractions (any *one*) : $3 \times 1 = 3$

   (i) $\dfrac{x}{(x+1)(x-2)^2}$

   (ii) $\dfrac{2x-3}{x^2+6x+8}$.

7. Answer any *two* of the following questions : $3 \times 2 = 6$

   (i) If 3rd and 7th terms of an Arithmetic Progression are 18 and 30 respectively, then find the progression.

   (ii) If a,b,c in A.P. and x,y,z are in G.P., then prove that $x^{b-c} y^{c-a} z^{a-b} = 1$.

(iii) By using properties of determinant find the value of

$$\begin{vmatrix} 1+x & y & z \\ x & 1+y & z \\ x & y & 1+z \end{vmatrix}.$$

8. Prove any *three* of the following : $2\times3=6$

(i) $\dfrac{\cos 5^0 + \sin 5^0}{\cos 5^0 - \sin 5^0} = \tan 50^0$

(ii) $\tan\left(\dfrac{\pi}{4} + \dfrac{\theta}{2}\right) = \sec\theta + \tan\theta$

(iii) $\tan^{-1}\dfrac{1}{2} + \tan^{-1}\dfrac{1}{3} = \dfrac{\pi}{4}$.

(iv) In any triangle ABC, $\dfrac{a+b}{c} = \dfrac{\cos\left(\dfrac{A-B}{2}\right)}{\sin\dfrac{C}{2}}$.

9. Answer any *two* of the following questions : $3\times2=6$

(i) If $A+B+C = \dfrac{\pi}{2}$, then prove that

$\tan A \tan B + \tan B \tan C + \tan C \tan A = 1$.

(ii) Solve : $2\cos^2 x + 3\cos x - 2 = 0, \ 0 \le x \le 2\pi$.

(iii) Show that $\cos^{-1}\left(\dfrac{12}{13}\right) + \sin^{-1}\left(\dfrac{3}{5}\right) = \sin^{-1}\left(\dfrac{56}{65}\right)$.

10. Answer any *two* of the following questions : $\qquad 3 \times 2 = 6$

(i) An irregular plot has the following offsets measured from one end at equal distance.

X	0	12	24	36	48	60	72	84	96	108	120
d	53	52	47	49	53	63	58	61	52	49	48

Find the area of the plot.

(ii) Find the whole surface area of a right prism whose height is 75 cm and whose base is a regular octagon of side 12cm.

(iii) A regular pyramid has a base area of 56 cm² and a volume of 224 cm³. What is the height of the pyramid ?

11. Answer the following questions : $\qquad 2+3+3=8$

(i) If the three points (3,0), (a, –2) and (8, 2) are collinear, then find the value of a.

(ii) Find the equation of a straight line passing through (–2, 4) and perpendicular to the line $4x + 5y + 2 = 0$.

(iii) Find the equations of two lines parallel to $5x - 12y + 26 = 0$ at a distance of 4 units from it.