Total No. of printed pages = 6

Sc-102/Math-I/1st Sem/2016/N

MATHEMATICS – I

Full Marks – 70

Pass Marks – 21

Time – Three hours

The figures in the margin indicate full marks for the questions.

GROUP – A

Answer any *seven* questions.

1. (a) Express in the form of A + iB

$$\frac{(2+3i)^2}{2+i}$$

(b) The pressure in a liquid varies as the depth when the density is constant and varies as the density when the depth is constant. The pressure is 1, when the depth is 32 and density is 1. Find the depth when the pressure is 3.4 units and density is 13.6 units.

2+3=5

Turn over

2. (a) For what value of m will the equation $x^2 - 2(5 + 2m)x + 3(7 + 10m) = 0$ have one root reciprocal of other ?

(b) Find the three terms in G.P whose product is 729 and the sum of their products in pairs is 819. 2+3=5

3. (a) A railway has 30 stations on its line. How many different single tickets of each class will be necessary ?

(b) Find the term independent of x in

$$\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$$

2+3=5

4. (a) If $\log_a^b = 10$, $\log_{6a}^{32b} = 5$, find a. 2+3=5

(b) Show that

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix} = abc\,(b - c)(c - a)(a - b)$$

2+3=5

5. (a) If $A^2 \alpha BC$, $B^2 \alpha CA$, $C^2 \alpha AB$, show that the product of the three constants of variation is unity.

(b) Show that 2+3=5

$$\frac{\frac{1}{\lfloor 2}+\frac{1}{\lfloor 4}+\frac{1}{\lfloor 6}+\ldots\ldots\ldots\ldots}{\frac{1}{\lfloor 1}+\frac{1}{\lfloor 3}+\frac{1}{\lfloor 5}+\ldots\ldots\ldots\ldots}=\frac{e-1}{e+1}$$

6. (a) Determine the value that K must have if the numbers 2K + 3, 3K+1 and 5K+3 are to form an A.P.

(b) Solve the equations by Cramer's Rule :

$$2x - 3y + z = -1$$

$$3x + y - 2z = 1$$

$$4x - y + z = 9$$

2+3=5

7. (a) If α and β be the roots of $ax^2 + bx + b = 0$ prove that

$$\sqrt{\frac{\alpha}{\beta}}+\sqrt{\frac{\beta}{\alpha}}+\sqrt{\frac{b}{a}}=0$$

(b) There are seven gentlemen and three ladies contesting for two vacancies, an elector can vote any number of candidates not exceeding the number of vacancies. In how many ways is it possible to vote ? 2+3=5

8. (a) Show that

$$\sqrt[lm]{\frac{x^{l}}{x^{m}}} \times \sqrt[mn]{\frac{x^{m}}{x^{n}}} \times \sqrt[nl]{\frac{x^{n}}{x^{l}}} = 1$$

(b) Prove that

$$\log 2 + 16 \log \frac{16}{15} + 12 \log \frac{25}{24} + 7 \log \frac{81}{80} = 1,$$

the base of the logarithm being 10. 2+3=5

9. (a) Show that $\arg \overline{z} = -\arg z$.

(b) Sum to n terms

$7 + 77 + 777 + ...$. 2+3=5

GROUP–B

10. Answer any *five* questions. 2×5=10

(a) (i) Express $55^{\circ}12'36''$ in centesimal measure.

(ii) Express $203^{\circ}\, 58''\, 73''$ in radians.

(b) Find the value of

cosec (-660°) and cot (-1575°).

(c) Show that

$\cos(60^{\circ} - A)\cos(30^{\circ} - B) - \sin(60^{\circ} - A)\sin(30^{\circ} - B) = \sin(A + B)$.

(d) Prove that $\sin 8\theta = 8 \sin\theta \cos\theta \cos 2\theta \cos 4\theta$

(e) Prove that $\frac{1+\cos A}{\sin A} = \cot \frac{A}{2}$

(f) Find the value of $\sin\left(\sin^{-1}\frac{1}{2} + \cos^{-1}\frac{1}{2}\right)$

(g) In any triangle the sides are $a = 7$, $b = 5$, $c = 8$. Find A.

11. Answer any *four* questions : 2½×4=10

(a) If $A+B+C = \pi$ and $\cos A = \cos B \cos C$, then prove that $\cot B \cot C = \frac{1}{2}$.

(b) Prove that

$$\cos^6\theta - \sin^6\theta = \cos 2\theta\left(1-\frac{1}{4}\sin^2 2\theta\right)$$

(c) Prove that

$$\cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ = \frac{1}{16}.$$

(d) Find the value of $\sin 18^\circ$.

(e) If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \frac{\pi}{2}$ show that $yz + zx + xy = 1$.

(f) $A+B+C = \pi$, prove that

$$\sin 2A + \sin 2B + \sin 2C = 4 \sin A \sin B \sin C$$

GROUP – C

12. Answer any *three* questions : $3 \times 5 = 15$

(a) A regular hexagon is inscribed in a circle of radius 5 cm. Find the area of the circle which is outside the hexagon.

(b) A river is 80 ft wide. Its depth at a distance of x ft from one bank is d ft and is given by the following table :

x :	0	10	20	30	40	50	60	70	80
d :	0	40	75	94	121	153	142	86	31

Find the cross-sectional area of the river.

(c) The area of the whole surface of a right circular cylinder is 3000 sq.cm and the diameter of the base is 40 cm. Find the volume and the height of the cylinder. $\left(\pi = \frac{22}{7}\right)$

(d) The perimeters of the ends of a frustum of a circular cone are 44 cm and 88 cm. If the height of the frustum is 16 cm, find its volume.

(e) Determine the volume of the pyramid whose height is $10\sqrt{7}$ feet and which stands on a triangle of sides 16 ft, 11 ft and 9 ft.