Total No. of printed pages = 10

END SEMESTER EXAMINATION–2022

Semester : 3rd (New)

Subject Code : Sc-303

MATHEMATICS – III

Full Marks – 70

Time – Three hours

The figures in the margin indicate full marks for the questions.

Instructions :

(i) *All* questions of PART–A are compulsory.

(ii) Answer any *five* questions from PART–B.

PART–A

Marks–25

1. Choose the correct answers : $1\times10=10$

(a) The order and degree of the differential equation $\frac{d^2y}{dx^2}-x\left(\frac{dy}{dx}\right)^2+2y=0$ is

(i) 2, 1 (ii) 1, 1

(iii) 1, 2 (iv) 2, 2

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(b) The differential equation whose general solution is $y = A\sin x + B\cos x$ is

(i) $\frac{d^2y}{dx^2} = y$ (ii) $\frac{d^2y}{dx^2} = -y$

(iii) $\frac{d^2y}{dx^2} = 0$ (iv) None of these

(c) The general solution of

$(x+2)dx + (y+3)dy = 0$ is

(i) $x^2 + 4x + 6y = 0$

(ii) $x^2 + y^2 + 4x + 6y = 0$

(iii) $x^2 + y^2 + 4x + 6y = c$

(iv) None of these

(d) In $\frac{dy}{dx} + \frac{1}{x}y = x^2$, integrating factor is

(i) x (ii) e^x

(iii) $\log x$ (iv) None of these

(e) The complementary function of

$(D^2+4)y = x^2$ is

(i) $C_1\cos 2x + C_2\sin 2x$

(ii) $C_1\cos x + C_2\sin x$

(iii) $C_1\cos 4x + C_2\sin 4x$

(iv) None of the above

(f) The Median of the distribution

7, 9, 5, 3, 10, 15, 21, 19, 17 is

(i) 12 (ii) 10

(iii) 15 (iv) 17

(g) If $u = \log(x^2+y^2)$ then $\frac{\partial u}{\partial y} =$

(i) $\frac{1}{x^2+y^2}$ (ii) $\frac{2y}{x^2+y^2}$

(iii) $\frac{x^2-y^2}{x^2+y^2}$ (iv) $\frac{2x}{x^2+y^2}$

(h) A coin is tossed successively three times. The probability of getting exactly one head is

(a) $\frac{1}{8}$ (ii) $\frac{1}{6}$

(iii) $\frac{3}{8}$ (iv) $\frac{2}{5}$

(i) The number of arbitrary constants in the general solution of

$5\frac{dy}{dx} + x^2 = 0$ is

(i) 1 (ii) 2

(iii) 3 (iv) 0

(j) Particular. Integral of the differential equation

$(D^2-3D+2)\,y = e^{3x}$ is

(i) $\frac{1}{12}e^{3x}$ (ii) $\frac{1}{2}e^{3x}$

(iii) $\frac{1}{3}e^{3x}$ (iv) None of these

2. Fill in the blanks : 1×5=5

(a) A differential equation $Mdx + Ndy = 0$.

(M, N are functions of x and y) is exact if and only if ———.

(b) Primitive of $xdx + ydy = 0$ is ———.

(c) The item with maximum frequency is called ———.

(d) Solution of $y = px + p^2$ is ———.

(e) A square matrix $A = (a_{ij})$ is called symmetric if ———.

3. Write true or false : 1×10=10

(a) $(AB)^T = B^T A^T$.

(b) If $y = f(x)$ be general solution of a differential equation and $y = p(x)$ be the particular integral of it then complementary function is $f(x) - p(x)$.

(c) 3rd order identity matrix is

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

(d) $(2x^2 + 4y)\,dx + (4x + y - 1)\,dy = 0$ is exact.

(e) Degree of the equation

$$\frac{d^2y}{dx^2} + xy\left(\frac{dy}{dx}\right)^4 + 2y = 2 \text{ is } 4.$$

(f) Characteristic equation of $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ is

$\lambda^2 - 7\lambda + 6 = 0.$

(g) The necessary and sufficient condition for a square matrix A to possess the inverse is that $|A| = 0$.

(h) If A is 2×2 matrix and B is 3×2 matrix then AB is a 2×2 matrix.

(i) Mode = 3 Median − 2 Mean.

(j) Auxiliary equation of $\frac{d^2y}{dx^2} + 9y = x^2$ is

$m^2 + 9x = 0.$

PART–B

Marks–45

4. (a) Find the order and degree of the following differential equation : 2+2=4

(i) $\frac{d^3y}{dx^3} + y = \sqrt{1+\frac{dy}{dx}}$.

(ii) $x\frac{dy}{dx} + \frac{5}{\frac{dy}{dx}} = x^3$.

(b) If $u = \log (x^3 + y^3 + z^3 - 3xyz)$ show that,

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}.$$ 5

5. Solve the following : 3×3=9

(a) $\frac{dy}{dx} + y\cot x = \cos x$.

(b) $(x+y+2)dx + (x-y+4)dy=0$.

(c) $\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 6y = 0$.

6. Solve the following : 3×3=9

(a) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 13y = e^{2x}$.

(b) $\frac{dy}{dx} = (x+y)^2$.

(c) $\frac{dy}{dx} = \frac{y}{x} + x^2$; given $y = 1$ when $x = 1$.

7. Solve the following : 3×3=9

(a) $\frac{d^2y}{dx^2} + y = x^3$.

(b) $(D^2 - 2D + 1)y = \cos 3x$.

(c) $y = px + p - p^2$.

8. (a) Calculate the Arithmetic mean and Standard deviation from the following data : 2+4=6

Marks	0–10	10-20	20-30	30-40	40-50
Frequency	8	13	16	8	5

(b) A bag contains 4 white balls and 2 black balls. Another bag contains 3 white balls and 5 black balls. If one ball is drawn from each bag, find the probability that 3

(i) both are white

(ii) both are black

(iii) one is white and one is black.

9. (a) Solve using matrix method : 5+4=9

$3x - y + 2z = -2$

$x + 2y + z = 1$

$5x + 3y + 2z = 4.$

(b) From the following data of marks in Mathematics and Statistics obtained by 6 students (out of total 50 marks) calculate the correlation coefficient :

Marks in Mathematics	35	30	28	29	13	45
Marks in Statistics	40	27	35	26	24	40

10. (a) Draw graph : 4+3=7

(i) $y = \sin x + \cos x \quad -\pi \le x \le \pi.$

(ii) $y = \tan x \quad -\pi/4 \le x \le \pi/4.$

(b) Find the transpose of the matrix : 2

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \\ 3 & 4 & 2 & 1 \end{bmatrix}.$$

11. (a) $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, show that 4

$A^2 - 4A - 5I = 0.$

(b) Find the maximum value of $z = 2x + 3y$ subject to the constraints 5

$x + y \leq 30,$

$0 \leq y \leq 12,$

$x - y \geq 0$

$y \geq 3$ and $0 \leq x \leq 20.$

12. (a) Calculate the Median and the Mean deviation from Median from the following data : 6

x :	20	18	16	14	12	10	8	6
f :	2	4	9	18	27	25	14	1

(b) Find the Mode of the distribution from the following data : 3

10-15	15-20	20-25	25-30	30-35	35-40	40-45
3	7	16	12	9	5	3