Total No. of printed pages = 10

#### **END SEMESTER EXAMINATION-2022**

Semester: 3rd (New)

Subject Code: Sc-303

### **MATHEMATICS – III**

Full Marks - 70

Time - Three hours

The figures in the margin indicate full marks for the questions.

## Instructions:

- (i) All questions of PART-A are compulsory.
- (ii) Answer any five questions from PART-B.

#### PART-A

Marks-25

1. Choose the correct answers:

 $1 \times 10 = 10$ 

(a) The order and degree of the differential

equation 
$$\frac{d^2y}{dx^2} - x\left(\frac{dy}{dx}\right)^2 + 2y = 0$$
 is

(i) 2, 1

(ii) 1, 1

(iii) 1, 2

(iv) 2, 2

(b) The differential equation whose general solution is  $y = A \sin x + B \cos x$  is

(i) 
$$\frac{d^2y}{dx^2} = y$$

(i) 
$$\frac{d^2y}{dx^2} = y$$
 (ii) 
$$\frac{d^2y}{dx^2} = -y$$

(iii) 
$$\frac{d^2y}{dx^2} = 0$$

- (iv) None of these
- (c) The general solution of

$$(x+2)dx + (y+3)dy = 0$$
 is

(i) 
$$x^2 + 4x + 6y = 0$$

(ii) 
$$x^2 + y^2 + 4x + 6y = 0$$

(iii) 
$$x^2 + y^2 + 4x + 6y = c$$

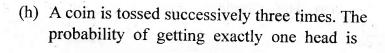
- (iv) None of these
- (d) In  $\frac{dy}{dx} + \frac{1}{x}y = x^2$ , integrating factor is
  - (i) x
- (ii) e<sup>x</sup>
- (iii) log x
- (iv) None of these

(e) The complementary function of

$$(D^2+4)y = x^2$$
 is

- (i)  $C_1\cos 2x + C_2\sin 2x$
- (ii)  $C_1 \cos x + C_2 \sin x$
- (iii)  $C_1\cos 4x + C_2\sin 4x$
- (iv) None of the above
- (f) The Median of the distribution

- (i) 12
- (ii) 10
- (iii) 15
- (iv) 17
- (g) If  $u = \log(x^2+y^2)$  then  $\frac{\partial u}{\partial y} =$
- (i)  $\frac{1}{x^2 + y^2}$  (ii)  $\frac{2y}{x^2 + y^2}$ (iii)  $\frac{x^2 y^2}{x^2 + y^2}$  (iv)  $\frac{2x}{x^2 + y^2}$



- (a)
- (ii)  $\frac{1}{6}$
- (iii)  $\frac{3}{8}$
- (iv)  $\frac{2}{5}$

(i) The number of arbitrary constants in the general solution of

$$5\frac{\mathrm{d}y}{\mathrm{d}x} + x^2 = 0 \quad \text{is} \quad$$

- (i) 1
- (ii) 2

(iii) 3

(iv) 0

Particular. Integral of the differential equation

$$(D^2-3D+2) y = e^{3x} is$$

- (i)  $\frac{1}{12}e^{3x}$  (ii)  $\frac{1}{2}e^{3x}$
- (iii)  $\frac{1}{3}e^{3x}$
- (iv) None of these

15/Sc-303/Maths-III (N)

2.	Fill	in	the	blanks	:

 $1 \times 5 = 5$ 

(a) A differential equation Mdx + Ndy = 0.

(M, N are functions of x and y) is exact if and only if ——.

- (b) Primitive of xdx + ydy = 0 is ———
- (c) The item with maximum frequency is called
- (d) Solution of  $y = px + p^2$  is ———.
- (e) A square matrix A =  $(a_{ij})$  is called symmetric if ———.
- 3. Write true or false:

 $1 \times 10 = 10$ 

- (a)  $(AB)^T = B^T A^T$ .
- (b) If y = f(x) be general solution of a differential equation and y = p(x) be the particular integral of it then complementary function is f(x) p(x).
- (c) 3rd order identity matrix is

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

15/Sc-303/Maths-III (N) (5)

(d) 
$$(2x^2 + 4y) dx + (4x + y - 1) dy = 0$$
 is exact.

(e) Degree of the equation

$$\frac{d^2y}{dx^2} + xy\left(\frac{dy}{dx}\right)^4 + 2y = 2 \text{ is } 4.$$

- (f) Characteristic equation of  $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$  is  $\lambda^2 7\lambda + 6 = 0$ .
- (g) The necessary and sufficient condition for a square matrix A to possess the inverse is that |A| = 0.
- (h) If A is 2×2 matrix and B is 3×2 matrix then AB is a 2×2 matrix.
- (i) Mode = 3 Median 2 Mean.
- (j) Auxiliary equation of  $\frac{d^2y}{dx^2} + 9y = x^2$  is  $m^2 + 9x = 0$ .

15/Sc-303/Maths-III (N) (6)

## PART-B

# Marks-45

4. (a) Find the order and degree of the following differential equation: 2+2=4

(i) 
$$\frac{d^3y}{dx^3} + y = \sqrt{1 + \frac{dy}{dx}}.$$

(ii) 
$$x \frac{dy}{dx} + \frac{5}{\frac{dy}{dx}} = x^3$$
.

(b) If  $u = \log (x^3 + y^3 + z^3 - 3xyz)$  show that,

$$\frac{\partial \mathbf{u}}{\partial \mathbf{x}} + \frac{\partial \mathbf{u}}{\partial \mathbf{y}} + \frac{\partial \mathbf{u}}{\partial \mathbf{z}} = \frac{3}{\mathbf{x} + \mathbf{y} + \mathbf{z}}.$$

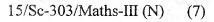
5. Solve the following:

$$3\times3=9$$

(a) 
$$\frac{dy}{dx} + y \cot x = \cos x$$
.

(b) 
$$(x+y+2)dx + (x-y+4)dy=0$$
.

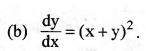
(c) 
$$\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 6y = 0$$
.



6. Solve the following:

$$3 \times 3 = 9$$

(a) 
$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 13y = e^{2x}$$
.





- (c)  $\frac{dy}{dx} = \frac{y}{x} + x^2$ ; given y = 1 when x = 1.
- Solve the following:

$$3 \times 3 = 9$$

(a) 
$$\frac{d^2y}{dx^2} + y = x^3$$
.

(b) 
$$(D^2 - 2D + 1)y = \cos 3x$$
.

(c) 
$$y = px + p - p^2$$
.

(a) Calculate the Arithmetic mean and Standard deviation from the following data: 2+4=6

Marks	0–10	10-20	20-30	30-40	40-50
Frequency	8	13	16	8	5

15/Sc-303/Maths-III (N)

2400(W)

- (b) A bag contains 4 white balls and 2 black balls. Another bag contains 3 white balls and 5 black balls. If one ball is drawn from each bag, find the probability that 3
  - (i) both are white
  - (ii) both are black
  - (iii) one is white and one is black.
- 9. (a) Solve using matrix method:

$$3x - y + 2z = -2$$

$$x + 2y + z = 1$$

$$5x + 3y + 2z = 4$$
.

(b) From the following data of marks in Mathematics and Statistics obtained by 6 students (out of total 50 marks) calculate the correlation coefficient:

Marks in Mathematics	35	30	28	29	13	45
Marks in Statistics	40	27	35	26	24	40

10. (a) Draw graph:

$$4+3=7$$

(i) 
$$y = \sin x + \cos x - \pi \le x \le \pi$$
.

(ii) 
$$y = \tan x - \frac{\pi}{4} \le x \le \frac{\pi}{4}$$
.

15/Sc-303/Maths-III (N) (9)

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \\ 3 & 4 & 2 & 1 \end{bmatrix}.$$

11. (a) 
$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$
, show that

$$A^2 - 4A - 5I = 0.$$



(b) Find the maximum value of z = 2x + 3y subject to the constraints 5

$$x+y \leq 30,$$

$$0 \leq y \leq 12,$$

$$x - y \ge 0$$

$$y \ge 3$$
 and  $0 \le x \le 20$ .

12. (a) Calculate the Median and the Mean deviation from Median from the following data: 6

x :	20	18	16	14	12	10	8	6
f :	2	4	9	18	27	25	14	1

(b) Find the Mode of the distribution from the following data:

10-15	15-20	20-25	25-30	30-35	35-40	40-45
3	7	16	12	9	5	3

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