Total No. of printed pages = 9

END SEMESTER EXAMINATION, NOVEMBER-2018

Semester : 3rd

Subject Code : Sc 303

MATHEMATICS – III

Full Marks – 70

Time – Three hours

The figures in the margin indicate full marks for the questions.

Instructions :

1. *All* the questions of PART – A are compulsory.
2. Answer any *five* questions from PART – B.

PART – A

Marks – 25

1. Fill in the blanks : $1\times10=10$

   (a) Order of the equation $\frac{d^4y}{dx^4}+4x\left(\frac{dy}{dx}\right)^2-6y=3$ is ——.

[Turn over

(b) Order of the equation $\frac{d^2y}{dx^2}+4\left(\frac{dy}{dx}\right)^4-y=2$ is ——.

(c) Degree of the equation

$\frac{d^2y}{dx^2}+xy\left(\frac{dy}{dx}\right)^4+2y=2$ is ——.

(d) Solution of (sec x) $\frac{dy}{dx}$ + y = 0 is ——.

(e) Solution of e^x $\frac{dy}{dx}$ + y = 0 is ——.

(f) Solution of x^2 y^2 $\frac{dy}{dx}$ + 4 = 0 is ——.

(g) Solution of $\sec^2$ x tan y dx + $\sec^2$ y tan x dy = 0 is ——.

(h) Primitive of $xdx+ydy=0$ is ——.

(i) Integrating factor of $\frac{dy}{dx}$ + sec x·y = tan x is ——.

(j) Integrating factor of $\frac{dy}{dx}+\frac{y}{x}$ = x is ..... .

2. Write true or false : 1×10=10

(a) $(x + y^2)\,dx + (2xy - e^y)\,dy = 0$ is an exact equation.

(b) Auxiliary equation of $\frac{d^2y}{dx^2}+9y=x^2$ is $m^2 + 9x = 0$.

(c) Solution of $y = px + p^2$ is $y = cx + c^2$.

(d) Formula for mode is $l+\frac{f-f_{-1}}{2f-f_{-1}-f_2}$

(e) Formula for median is

$$\sqrt{\frac{1}{N}\sum fd^2-\left(\frac{1}{N}\sum fd\right)^2}$$

(f) Quartile deviation is a measure of central tendency.

(g) Mean of 4, 6, 1, 11, 3, 5, 8, 2 is 8.

(h) When tossing a coin getting a head and getting a tail are mutually exclusive events.

(i) Probability of drawing a red ball from a box containing 8 black balls and 2 red balls is $\frac{1}{4}$.

(j) Probability of drawing a Red Queen from a pack of cards is $\frac{4}{52}$.

3. Choose the correct answer : 1×5=5

(a) Transpose of $\begin{pmatrix} 1 & 0 & -2 \\ 3 & -1 & 4 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 2 \\ 1 & 3 & -5 \end{pmatrix}$ is

(i) $\begin{pmatrix} 1 & 4 \\ 0 & 2 \\ 0 & -1 \end{pmatrix}$ (ii) $\begin{pmatrix} 3 & 1 \\ 0 & -2 \\ 1 & 1 \end{pmatrix}$

(iii) $\begin{pmatrix} 1 & -2 \\ -1 & 2 \\ 0 & 1 \end{pmatrix}$ (iv) $\begin{pmatrix} 1 & 0 & 2 \\ 1 & 4 & 1 \end{pmatrix}$

(b) 3rd order identity matrix is

(i) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{pmatrix}$ (ii) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$

(iii) $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ (iv) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(c) Characteristic equation of $\begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$ is

(i) $\lambda^3+3\lambda-1=0$

(ii) $(\lambda-1)^2(\lambda-5)=0$

(iii) $\lambda^3+3\lambda^2+\lambda 5-1=0$

(iv) $\lambda^2+2\lambda+7=0$

(d) (4, 0) is a solution of

(i) $2x - y < 1$

(ii) $x + y \geq 6$

(iii) $x + y \geq 4$

(iv) $x + y > 4$

(e) (1, 6) is a solution of

(i) $x - y < 1$ (ii) $2x - 3y \geq 6$

(iii) $x + y \geq 8$ (iv) $3x + y > 9$

PART – B

Marks – 45

4. (a) Find order and degree of the following differential equations : 2+2=4

(i) $\frac{d^2y}{dx^2} - 2\left(\frac{dy}{dx}\right)^2 + y = 0$

(ii) $x\left(\frac{d^3y}{dx^3}\right)^3 - x^2\left(\frac{dy}{dx}\right)^4 + y^6 = 0$

(b) Given $f(x, y) = 5x^4 + 3x^2y - e^{x/y} \sin y$.

Find

(i) $\frac{\partial f}{\partial x}$ (ii) $\frac{\partial f}{\partial y}$ 2+3=5

5. Solve the following : 3×3=9

(a) $\frac{dx}{x} = \sin y \, dy$

(b) $(e^y + 3) \sin x \, dx + e^y \cos x \, dy = 0$

(c) $(2x + y + 3) \, dy = (2x + y - 3) \, dx$

6. Solve the following : 3×3=9

(a) $\frac{dy}{x} + 2xy = 2e^{-x^2}$

(b) $\frac{dy}{dx} + \tan x \tan y = \cos x \sec y$

(c) $(2x - y) \, dx = (x - y) dy$

7. Solve the following : 3×3=9

(a) $\frac{d^3y}{dx^3} + y = 0$

(b) $\frac{d^2y}{dx^2} - y = 0$, if $y = 0$, $\frac{dy}{dx} = 0$ at $x = 0$

(c) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 6e^{3x} + 7e^{-2x}$