Total No. of printed pages = 10

Sc-303/Maths-III/3rd Sem/2018/M

MATHEMATICS – III

Full Marks – 70

Pass Marks – 28

Time – Three hours

The figures in the margin indicate full marks for the questions.

PART – A

1. Choose the correct answer : $1\times10=10$

(a) The degree of the differential equation

$$\sqrt{1+\frac{d^2y}{dx^2}}=\frac{dy}{dx}$$ is

(i) 2 (ii) 1

(iii) 3 (iv) 4

[Turn over

(b) The order of the differential equation

$$\frac{d^3y}{dx^3} = 3 \text{ is}$$

(i) 1 (ii) 2

(iii) 3 (iv) 4

(c) The name of the equation $y = px + f(p)$, $p = \frac{dy}{dx}$

(i) Linear differential equation

(ii) Exact differential equation

(iii) Clairant's differential equation

(iv) Bernoullie's differential equation.

(d) If $y = Ax^2 + Bx + C$, A, B, C are constants, how many times you have to differentiate to make a differential equation ?

(i) 1 (ii) 2

(iii) 3 (iv) None of these

(e) In $\frac{dy}{dx} + Py = Q$, P, Q are functions of x or constants. What is the integrating factor ?

(i) $\int Pdx$ (ii) $\int Qdx$

(iii) $e^{\int Pdx}$ (iv) None of these

(f) If f (x, y) is a function of x, y, then in $\frac{\partial}{\partial x} f(x, y)$

(i) x is considered as a constant

(ii) y is considered as a constant

(iii) both x, y are considered as constant

(iv) None of these

(g) Integrating factor of $\frac{dy}{dx} + \frac{y}{x} = x^2$ is

(i) x^2 (ii) sin x

(iii) e^x (iv) x

(h) How many solutions are there for the equation $y = px + f(p)$, $p = \frac{dy}{dx}$?

(i) 1 (ii) 2

(iii) 3 (iv) None of these

(i) In $\frac{d^2y}{dx^2} + \frac{Pdy}{dx} + Qy = 0$ the solution consists of how many parts ?

(i) 1 (ii) 2

(iii) 3 (iv) None of these

(j) In $\frac{d^2y}{dx^2} + \frac{Pdy}{dx} + Qy = 0$ the auxiliary equation is

(i) quadratic (ii) cubic

(iii) bi-quadratic (iv) None of these.

2. Write true or false : 1×5=5

(i) The equation $\sin x = \cos x$ has only one solution in $0 \leq x \leq \pi/2$.

(ii) To find the solution of the equation $f(x) = 0$ we divide $f(x)$ into linear and non-linear parts.

(iii) In $x^3 + 2x - 20 = 0$, $y = x^3$ is the non-linear part and $y = 20 - 2x$ is the linear part.

(iv) To draw a straight line two points are necessary and sufficient.

(v) To draw a curve a minimum of 10 to 12 points are necessary.

3. Answer the following questions : 1×5=5

(i) Ascending means gradually increasing / gradually decreasing.

(ii) What are the measures of dispersion ?

(iii) What are the minimum and maximum values of the correlation coefficient r ?

(iv) When correlation coefficient $r = 0$, what type of correlation is there ?

(v) To calculate median how the data should be arranged ?

4. Fill up the blanks : $1 \times 5 = 5$

(a) Distance between (1, 2, 1) and (3, 4, –5) is ———.

(b) The position vector of A (9, 0, 2) is ———.

(c) The unit vector is defined as ———.

(d) Dot product of $\hat{i} + 7\hat{j} - \hat{k}$ and $2\hat{i} - 3\hat{j} + 5\hat{k}$ ———.

(e) When the dot product of two vectors are '0' (zero), then the vectors are ——— .

PART – B

5. (a) Form a differential equation from $y = mx$. 2

(b) Solve any *four* equations : $3 \times 4 = 12$

(i) $(1 - x^2)\, y\, dy + (1 - y^2)\, x\, dx = 0$

(ii) $\frac{dy}{dx} + y \cot x = \text{cosec } x$

(iii) $p^2 - 5p + 6 = 0,\ p = \frac{dy}{dx}$

(iv) $\frac{dy}{dx} = \frac{x + y + 1}{2x + 2y - 3}$

(v) $(x + y + 1)dx + (x + y + 2)dy = 0$

(vi) $\frac{dy}{dx} = \frac{x^3 + y^3}{xy^2}$

(c) Solve any *one* : 4

(i) $\frac{d^2y}{dx^2} - \frac{5dy}{dx} + 6y = 0$

(ii) $\frac{d^2y}{dx^2} + y = \cos x$

6. Answer any *two* questions : 5×2=10

(i) Solve graphically $x^3 - 3x - 12 = 0$

(ii) Draw the graph of $y = \cos x$, $0 \leq x \leq \pi/2$

(iii) The given law is $y = a + bx^2$ and the data is

x :	0	2	4	6	8	10
y =	7.76	11.8	24.4	43.6	71.2	107.0

Find best values of a and b.

4

7. Answer any *three* questions : 4×3=12

(i) Calculate mean, median and mode for the following data :

Class interval	frequency
0 – 10	6
10 – 20	5
20 – 30	18
30 – 40	15
40 – 50	7

(ii) Calculate coefficient of correlation for X and Y :

X :	65	66	67	67	68	69	70	72
Y :	67	68	65	68	72	72	69	71

(iii) Calculate standard deviations :

Class interval	frequency
15 – 20	2
20 – 25	2
25 – 30	7
30 – 35	10
35 – 40	6
40 – 45	2

(iv) Calculate median, Q_1 and Q_3 from the following data :

Wages (in Rs.)	No. of workers
Less than 50	3
Less than 100	8
Less than 150	12
Less than 200	17
Less than 250	32
Less than 300	38
Less than 350	45
Less than 400	50

8. Answer any *one* question : 3+2=5

(i) (a) Find the disection cosines of the line joining (3, –1, 1) and (–2, –3, –1)

(b) If $\bar{a} = \hat{i} - \hat{j} + \hat{k}$ and $\bar{b} = 2\hat{i} + \hat{j} - \hat{k}$, find $|\bar{a} + \bar{b}|$ and $\bar{a}.\bar{b}$.

(ii) (a) If a straight line makes equal angles X, Y, Z axes, find the direction cosines of the lines.

(b) If $\bar{a} = 3\hat{i} - \hat{j} - 4\hat{k}$ and $\bar{b} = 2\hat{i} + \hat{j} - 3\hat{k}$, find the unit vector perpendicular to the plane of $\bar{a}$ and $\bar{b}$.