Total No. of printed pages = 10

Sc-303/Maths-III/3rd Sem/2017/N

MATHEMATICS – III

Full Marks – 70

Time – Three hours

The figures in the margin indicate full marks for the questions.

PART – A

1. Choose the correct answer : 1×10=10

(a) The degree of the equation

$$\frac{d^2y}{dx^2}-\left(\frac{dy}{dx}\right)^2+3y=0$$ is

(i) 2 (ii) 1 (iii) 3 (iv) 4

(b) The degree of the equation

$$\left(\frac{d^2y}{dx^2}\right)^2+\frac{dy}{dx}+5y=0$$ is

(i) 2 (ii) 1 (iii) 3 (iv) 4

[Turn over

(c) The order of the equation

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]\frac{d^3y}{dx^3}-3\frac{dy}{dx}\left(\frac{d^2y}{dx^2}\right)^2=0 \quad \text{is}$$

(i) 2 (ii) 1

(iii) 3 (iv) 4

(d) Primitive of $xdx + ydy = 0$

(i) $x^2 = 2y$ (ii) $y^2 = x$

(iii) $x + y = c$ (iv) $x^2 + y^2 = c$

(e) Primitive of $\frac{d^2y}{dx^2}+4y=0$

(i) $x = y$

(ii) $y = e^{2x}$

(iii) $y = (A + Bx)e^{2x}$

(iv) $y = A \cos 2x + B \sin 2x$

(f) Condition of exactness of the equation $Mdx + Ndy = 0$ is

(i) $\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 1$

(ii) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

(iii) $\frac{\partial M}{\partial y} + \frac{\partial N}{\partial x} = 0$

(iv) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$

(g) The Integrating Factor of $\frac{dy}{dx} + \frac{y}{x} = x^2$ is

(i) x^2 (ii) $\sin x$

(iii) e^x (iv) x

(h) The Integrating Factor of $\frac{dy}{dx} - \frac{2xy}{1-x^2} = x$ is

(i) $1 - x^2$ (ii) $\frac{1}{\sqrt{1-x^2}}$

(iii) $\sin^{-1} x$ (iv) $\sqrt{1-x^2}$

(i) Complementary Function of

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = \sin x$$

(i) $c_1e^x + c_2e^{-x}$ (ii) $(c_1 + c_2x)\,e^x$

(iii) $\sin x$ (iv) $e^{\sin x}$

(j) Particular Integral of $\frac{d^2y}{dx^2} + y = 5e^{2x}$

(i) $\frac{5}{9}e^{2x}$ (ii) $\frac{1}{9}e^{2x}$

(iii) e^{5x} (iv) $9e^{2x}$

2. Write true or false : $1\times5=5$

(i) Real root of the equation $x^3 + 2x - 20 = 0$ is the x-coordinate of the point of intersection of the graphs $y = x^3$ and $y = 20 - 2x$.

(ii) Roots of $x^2 + 3x + 2 = 0$ are the x-coordinate of the point at which the curve meets the x-axis.

(iii) $x^3 = 1$ has three real roots.

(iv) $\sin x = \cos x$ has infinite number of roots.

(v) The law $y = a + bx^2$ can be found from a set of observations (x, y) by transformation $Y = a + bX$ where $\sqrt{x} = X$.

3. Answer the following questions : 1×5=5

(a) What are the three measures of central tendency ?

(b) For the observations : 23, 13, 54, 34, 25, 32, 39 ; what is the median ?

(c) Write the formula for variance.

(d) Find the probability of drawing a king from a pack of cards.

(e) Three coins are tossed together. Write down the sample space.

4. Fill in the blanks : 1×5=5

(a) Distance between (1, 2, – 1) and (0, 2, 1) is ———.

(b) —— is the position vector of A(3, 0, 5).

(c) Direction cosines of a line parallel to x-axis are ——.

(d) Dot product of $4i + 7j - 2k$ and $i + 3j - k$ is ——.

(e) —— is the unit vector parallel to $i + j + k$.

PART – B

5. (a) Form a differential equation whose primitive is $y = mx^3 + 2$ 2

Or

Solve : $xdx + ydy = 0$

(b) Solve any *four* questions : $3 \times 4 = 12$

(i) $x\frac{dy}{dx} + \cot y = 0$, given $y = \frac{\pi}{4}$, $x = \sqrt{2}$

(ii) $x\frac{dy}{dx} - 3y = x^2$

(iii) $(x + y)^2 \frac{dy}{dx} = 4$

(iv) $\frac{dy}{dx} = \frac{y}{x} + \cot\frac{y}{x}$

(v) $(y - x \sin x)\, dx + (x - 2e^y)\, dy = 0$

(vi) $xdx + ydy + \frac{xdy - ydx}{x^2 + y^2} = 0$

(vii) $y = px + p - p^2$

(c) Solve any *one* : 4

(i) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^{2x} + x^2 + x$

(ii) $\frac{dy}{dx} = \frac{y - x + 1}{y + x + 5}$

6. Answer any *two* questions : $5 \times 2 = 10$

(i) Fit the linear law $P = aW + b$ to the following data :

P :	60	70	90	100	125
W :	225	270	380	430	550

Hence find the value of a and b.

(ii) Solve graphically : $x^2 + 5x - 6 = 0$

(iii) The following values of x and y obey the law $y = ae^{bx}$. Find a and b.

x :	2.70	2.87	3.26	3.68	3.89
y :	3.86	4.2	5.1	6.3	7

(iv) Solve graphically : $x^3 = 111$

7. Answer any one question :

(i) Find Mean and Median from the following data :

Class Interval	Frequency
0 – 10	3
10 – 20	5
20 – 30	7
30 – 40	9
40 – 50	4
50 – 60	3
60 – 70	4

(ii) Find Standard deviation from the following data :

Class Interval	Frequency
0 – 5	13
5 – 10	17
10 – 15	19
15 – 20	25
20 – 25	21
25 – 30	18
30 – 35	16
35 – 40	15
40 – 45	14
45 – 50	10

7. Answer any *three* questions : $4 \times 3 = 12$

(i) Find Mean and Median from the following data :

Class Interval	Frequency
0 – 10	3
10 – 20	5
20 – 30	7
30 – 40	9
40 – 50	4
50 – 60	3
60 – 70	4

(ii) Find Standard deviation from the following data :

Class Interval	Frequency
0 – 5	13
5 – 10	17
10 – 15	19
15 – 20	25
20 – 25	21
25 – 30	18
30 – 35	16
35 – 40	15
40 – 45	14
45 – 50	10

(iii) The following are [illegible] students in a class in Mathematics and Physics :

Mathematics:	80	45	55	56	58	60	63	68	70	75
Physics:	81	56	50	48	60	62	64	65	70	74

Compute coefficient of correlation.

(iv) Find Mode from the following data :

Marks	Number of students
Below 10	3
Below 20	10
Below 30	13
Below 40	20
Below 50	37
Below 60	42
Below 70	45
Below 80	48
Below 90	50

(v) From a pack of cards two cards are drawn at random. Find the probability that

(a) they are kings.

(b) they are red cards.

(iii) The following are the marks obtained by 10 students in a class in Mathematics and Physics :

Mathematics:	80	45	55	56	58	60	63	68	70	75
Physics:	81	56	50	48	60	62	64	65	70	74

Compute coefficient of correlation.

(iv) Find Mode from the following data :

Marks	Number of students
Below 10	3
Below 20	10
Below 30	13
Below 40	20
Below 50	37
Below 60	42
Below 70	45
Below 80	48
Below 90	50

(v) From a pack of cards two cards are drawn at random. Find the probability that

(a) they are kings.

(b) they are red cards.

8. Answer any *one* question : 3+2=5

(i) (a) Find the direction cosines of the line joining (1, 0, 7) and (4, 3, 2).

(b) Find the ratio in which the line joining the points (2, 4, 5) and (– 3, 5, – 4) is divided by the x-plane.

(ii) (a) If $a = i - 3j + 3k$ and $b = 2i - 4j + k$, find $(a + b) \times (a - b)$.

(b) Show that $2i - j + k$ and $i - 3j - 5k$ are mutually perpendicular.