END SEMESTER/RETEST EXAMINATION 2021

Semester : 2nd

Subject Code : SC- 202

Subject : MATHEMATICS-II

Full Marks : 70

Duration : 03 hours

Instruction :

All questions in PART- A are compulsory.

PART – A

Marks – 25

1. **Fill in the blanks :** $1\times10=10$

(a) For a rectangular hyperbola the value of eccentricity (e) is _____.

(b) Focus of the parabola $x^2=y$ is _____.

(c) The condition that the two lines having direction cosines l_1,m_1,n_1 and l_2,m_2,n_2 are perpendicular is _____.

(d) The centre of the circle $x^2+y^2+2x-6y=0$ is _____.

(e) If for a conic section $e<1$, then it is called _____.

(f) $\lim\limits_{x\to0}\dfrac{\sin 2x}{x}=$ _____.

(g) A function $f(x)$ is said to be _____ if $f(-x)=f(x)$ for all x.

(h) The domain of $f(x)=\dfrac{1}{x(x+3)}$ is _____.

(i) The 2nd derivative of $y=\dfrac{1}{2}e^{5x}$ is _____.

(j) $\int\limits_{-a}^{a}f(x)dx=0$, if $f(x)$ is _____ function.

2. **Choose the correct answer :** $1\times10=10$

(a) The magnitude of $\vec{r}=\hat{i}-3\hat{j}+5\hat{k}$ is

(i) 35 (ii) $\sqrt{32}$ (iii) 32 (iv) $\sqrt{35}$

(b) The unit vector parallel to $5\hat{i}+2\hat{j}$ is

(i) $\frac{5\hat{i}+2\hat{j}}{\sqrt{29}}$ (ii) $\pm\frac{5\hat{i}+2\hat{j}}{29}$ (iii) $\pm\frac{5\hat{i}+2\hat{j}}{\sqrt{29}}$ (iv) None of these

(c) The value of $\int x^3dx$ is

(i) 1 (ii) 0 (iii) 3 (iv) None of these

(d) If $f(x)=\log(4x+5)$, then the value of $f'(0)$ is

(i) $\frac{4}{5}$ (ii) $\frac{1}{5}$ (iii) 0 (iv) 1

(e) $\int x^n dx=\frac{x^{n+1}}{n+1}$ is valid for

(i) $n=1$ (ii) $n\neq1$ (iii) $n\neq-1$ (iv) For all values of n

(f) Direction ratios of the line joining the points (1, -2, 3) and (2, 3, -4) are

(i) 1, 5, 6 (ii) 2, 5, -7 (iii) 1, 0, 6 (iv) 1, 5, -7

(g) The value of $\frac{d^2}{dt^2}(t^3+\sin t)$ at $t=0$ is

(i) 1 (ii) -1 (iii) $\frac{1}{2}$ (iv) None of these

(h) The tangent to the circle $x^2+y^2=13$ at $(-2,-3)$ is

(i) $2x+3y+13=0$ (ii) $3x+2y-13=0$ (iii) $x+3y-13=0$ (iv) None of these

(i) If $f(x)=x^2, g(x)=\cos x$, then $f\{g(x)\}$ is

(i) $\cos^2 x$ (ii) $\cos x^2$ (iii) $x^2\cos x$ (iv) None of these

(j) The derivative of $y=\sqrt{ax^2+bx+c}$ with respect to x is

(i) $\frac{ax+b}{y}$ (ii) $\frac{2ax+b+c}{y}$ (iii) $2ax+b$ (iv) $\frac{2ax+b}{2y}$

3. **Write True or False :** $1\times5=5$

(i) Direction Cosines of Z- axis are $1, 0, 0$.

(ii) Definite integral can be expressed as the limit of a sum .

(iii) The maximum value of a function can not be less than its minimum value.

(iv) The latusrectum of $\frac{x^2}{16}-\frac{y^2}{25}=1$ is $\frac{25}{2}$.

(v) The value of $\int_0^1 e^{-x}dx$ is $1-\frac{1}{e}$.

PART- B

4. **Answer any two questions :** $2\times3=6$

(a) Find the equation of the circle whose centre is (4,5) and which passes through the centre of the circle $x^2+y^2+4x-6y-12=0$.

(b) Find the focus, equation to the directrix and the length of the latusrectum of the parabola $x^2+10y=0$.

(c) Prove that the line $3x-2y=8$ touches the ellipse $3x^2+4y^2=16$.

5. **Answer any one question :**

(a) (i) Find the ratio in which the line segment joining the points (3,5,7) and (-2,4,6) is divided by YZ- plane. 2

(ii) If $\vec{a}=(4,2,-3), \vec{b}=(-1,1,1)$, and $\vec{c}=(0,3,1)$, find $\vec{a}.(\vec{b}\times\vec{c})$. 3

Or

(b) (i) The angle between two lines having Direction ratios a , 0 , 3 and -2 , -1 , 4 is 90° . Find the value of a . 2

(ii) A particle is acted on by a constant force $\hat{i}+2\hat{j}-6\hat{k}$ and is displaced from the point (2 , 1 , -1) to (3 , 4 , 5). Find the amount of work done by the force. 3

6. Examine the continuity of the following function at $x=1$ 2

$$f(x)=\begin{cases} x, & 0\le x\le 1 \\ 2-x, & 1<x<2 \end{cases}$$

7. Evaluate the limits (Any two): $2\times3=6$

(a) $\lim\limits_{x\to0}\dfrac{e^{2x}-e^{3x}}{x}$ (b) $\lim\limits_{x\to0}\dfrac{\sqrt{1+x}-\sqrt{1-x}}{x}$

(c) $\lim\limits_{x\to\infty}\dfrac{2x^2-3x+1}{3x^2+2x-1}$

8. Find $\dfrac{dy}{dx}$ (Any two) : $2\times3=6$

(a) $x^y=y^x$ (b) $x=2t-1, y=t^2$ (c) $y=x\sin^{-1}x$

9. If $y=A\cos nx+B\sin nx$, prove that $\dfrac{d^2y}{dx^2}+n^2y=0$ 2

10. Find the maximum and minimum values of $f(x)=x+\dfrac{1}{x}$. 3

11. **Integrate (Any three) :** $3\times3=9$

(a) $\int\left(x+\dfrac{1}{x}\right)^3dx$ (b) $\int\dfrac{e^{2x}}{e^x+1}dx$ (c) $\int\dfrac{dx}{(2+x)\sqrt{1+x}}$

(d) $\int_0^1 xe^x dx$ (e) $\int_0^1\dfrac{\sin^{-1}x}{\sqrt{1-x^2}}dx$

12. (a) Using property of definite integral prove that $\int_0^{\frac{\pi}{2}}\log\tan x\,dx=0$ 3

(b) Using definite integral find the area of the region bounded by the parabola $y^2=8x$ and its latusrectum. 3

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