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# END SEMESTER/RETEST EXAMINATION 2021

Semester : 2<sup>nd</sup>

Subject Code : SC- 202

# Subject : MATHEMATICS-II

## Full Marks: 70

### **Duration : 03 hours**

**Instruction** :

All questions in PART- A are compulsory.

### PART-A

#### Marks - 25

#### 1. Fill in the blanks :

 $1 \times 10 = 10$ 

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(a) For a rectangular hyperbola the value of eccentricity (e) is \_\_\_\_\_\_.

- (b) Focus of the parabola  $x^2 = y$  is \_\_\_\_\_.
- (c) The condition that the two lines having direction cosines  $l_1, m_1, n_1$  and  $l_2, m_2, n_2$  are perpendicular is \_\_\_\_\_\_.

(d) The centre of the circle  $x^2 + y^2 + 2x - 6y = 0$  is \_\_\_\_\_.

(e) If for a conic section e < 1, then it is called \_\_\_\_\_\_.

- (f)  $\lim_{x \to 0} \frac{\sin 2x}{x} = \underline{\qquad}$
- (g) A function f(x) is said to be \_\_\_\_\_ if f(-x) = f(x) for all x.
- (h) The domain of  $f(x) = \frac{1}{x(x+3)}$  is \_\_\_\_\_.
- (i) The 2<sup>nd</sup> derivative of  $y = \frac{1}{2}e^{5x}$  is \_\_\_\_\_.
- (j)  $\int_{-\alpha}^{\alpha} f(x) dx = 0$ , if f(x) is \_\_\_\_\_\_ function.

# 2. Choose the correct answer :

- (a) The magnitude of  $\vec{r} = \hat{i} 3\hat{j} + 5\hat{k}$  is
  - (i) 35 (ii)  $\sqrt{32}$  (iii) 32 (iv)  $\sqrt{35}$

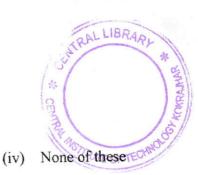
(b) The unit vector parallel to  $5\hat{i} + 2\hat{j}$  is

- (i)  $\frac{5\hat{i}+2\hat{j}}{\sqrt{29}}$  (ii)  $\pm \frac{5\hat{i}+2\hat{j}}{29}$  (iii)  $\pm \frac{5\hat{i}+2\hat{j}}{\sqrt{29}}$
- (c) The value of  $\int x^3 dx$  is
  - (i) 1 (ii) 0 (iii) 3 (iv) None of these
- (d) If  $f(x) = \log(4x+5)$ , then the value of f'(0) is
  - (i)  $\frac{4}{5}$  (ii)  $\frac{1}{5}$  (iii) 0 (iv) 1
- (e)  $\int x^n dx = \frac{x^{n+1}}{n+1}$  is valid for (i) n = 1 (ii)  $n \neq 1$  (iii)  $n \neq -1$  (iv) For all values of n(f) Direction ratios of the line joining the points (1, -2, 3) and (2, 3, -4) are (i) 1, 5, 6 (ii) 2, 5, -7 (iii) 1, 0, 6 (iv) 1, 5, -7 (g) The value of  $\frac{d^2}{dt^2}(t^3 + \sin t)$  at t = 0 is
  - (i) 1 (ii) -1 (iii)  $\frac{1}{2}$  (iv) None of these

(h) The tangent to the circle  $x^2 + y^2 = 13$  at (-2,-3) is

(i) 2x+3y+13=0 (ii) 3x+2y-13=0 (iii) x+3y-13=0 (iv) None of these

(i) If 
$$f(x) = x^2$$
,  $g(x) = \cos x$ , then  $f\{g(x)\}$  is  
(i)  $\cos^2 x$  (ii)  $\cos x^2$  (iii)  $x^2 \cos x$  (iv) None of these



 $1 \times 10 = 10$ 

(i) The derivative of  $y = \sqrt{ax^2 + bx + c}$  with respect to x is

(i) 
$$\frac{ax+b}{y}$$
 (ii)  $\frac{2ax+b+c}{y}$  (iii)  $2ax+b$ 

# 3. Write True or False :

- (i) Direction Cosines of Z-axis are 1, 0, 0.
- (ii) Definite integral can be expressed as the limit of a sum .
- (iii) The maximum value of a function can not be less than its minimum value.

(iv) The latustrectum of 
$$\frac{x^2}{16} - \frac{y^2}{25} = 1$$
 is  $\frac{25}{2}$ .

(v) The value of 
$$\int_{0}^{1} e^{-x} dx$$
 is  $1 - \frac{1}{e}$ .

### PART-B

### 4. Answer any two questions :

- (a) Find the equation of the circle whose centre is (4,5) and which passes through the centre of the circle  $x^2 + y^2 + 4x 6y 12 = 0$ .
- (b) Find the focus, equation to the directrix and the length of the latusrectum of the parabola  $x^2 + 10y = 0$ .

(c) Prove that the line 3x - 2y = 8 touches the ellipse  $3x^2 + 4y^2 = 16$ .

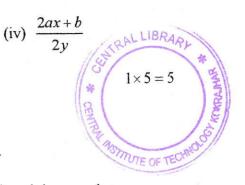
# 5. Answer any one question :

(a) (i) Find the ratio in which the line segment joining the points (3,5,7) and (-2,4,6) is divided by YZ- plane.

(ii) If 
$$\vec{a} = (4,2,-3), \vec{b} = (-1,1,1)$$
, and  $\vec{c} = (0,3,1)$ , find  $\vec{a} \cdot (\vec{b} \times \vec{c})$ .

Or

(b) (i) The angle between two lines having Direction ratios a, 0, 3 and -2, -1, 4 is 90°. Find the value of a.



 $2 \times 3 = 6$ 

(ii) A particle is acted on by a constant force  $\hat{i} + 2\hat{j} - 6\hat{k}$  and is displaced from the point (2, 1, -1) to (3, 4, 5). Find the amount of work done by the force. 3

6. Examine the continuity of the following function at x = 1

$$f(x) = \begin{cases} x, & 0 \le x \le 1 \\ 2 - x, & 1 < x < 2 \end{cases}$$

7. Evaluate the limits (Any two):

(a) 
$$\lim_{x \to 0} \frac{e^{2x} - e^{3x}}{x}$$
 (b)  $\lim_{x \to 0} \frac{\sqrt{1 + x} - \sqrt{1 - x}}{x}$ 

(c) 
$$\lim_{x \to \infty} \frac{2x^2 - 3x + 1}{3x^2 + 2x - 1}$$

11. Integrate (Any three):

8. Find  $\frac{dy}{dx}$  (Any two):  $2 \times 3 = 6$ 

(a) 
$$x^y = y^x$$
 (b)  $x = 2t - 1, y = t^2$  (c)  $y = x \sin^{-1} x$ 

9. If  $y = A\cos nx + B\sin nx$ , prove that  $\frac{d^2y}{dx^2} + n^2y = 0$ 

10. Find the maximum and minimum values of  $f(x) = x + \frac{1}{x}$ .

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 $2 \times 3 = 6$ 

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 $3 \times 3 = 9$ 

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(a)  $\int \left(x + \frac{1}{x}\right)^3 dx$  (b)  $\int \frac{e^{2x}}{e^x + 1} dx$  (c)  $\int \frac{dx}{(2+x)\sqrt{1+x}}$ (d)  $\int_0^1 xe^x dx$  (e)  $\int_0^1 \frac{\sin^{-1}x}{\sqrt{1-x^2}} dx$ 

12. (a) Using property of definite integral prove that  $\int_{0}^{2} \log \tan x \, dx = 0$ 

(b) Using definite integral find the area of the region bounded by the parabola  $y^2 = 8x$  and its latusrectum.

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