Total No. of printed pages = 8

MATHEMATICS–II

Full Marks – 70

Time – Three hours

The figures in the margin indicate full marks for the questions.

PART–A

1. Match the following : 5

(i) $\lim_{x \to 0} \frac{x^2 - 4}{x - 2}$	2
(ii) $\lim_{x \to 0} \frac{\sin 2x}{x}$	cos x
(iii) $\frac{d}{dx}(x^2 - 3)\Big\|_{x=1}$	2
(iv) $\lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}$	0
(v) $\frac{d}{dx}\cos^2 x\Big\|_{x=0}$	4

[Turn over

2. Fill in the blanks : 15

(i) Locus of a point moving at equal distance from two given points P and Q is a .... and is the perpendicular bisector of PQ.

(ii) The major axis of the ellipse $\frac{x^2}{2}+\frac{y^2}{9}=1$ is .....

(iii) The point of discontinuity of the function $f(x) = \frac{x^2-4}{x+2}$ is .....

(iv) The .... order derivative of $y = e^{4x}$ is $16e^{4x}$.

(v) Domain of $f(x) = \frac{x-1}{x(x+2)}$ is .....

(vi) If $f(x) = \cos 2x$, $0 < x < \pi$, then $f\left(\frac{\pi}{4}\right)$ = ...

(vii) ..... is the maximum value of $y = \sin x$ in $-\pi < x < \pi$.

(viii) The value of $\lim_{x\to\infty} \frac{\sin x}{x}$ is .....

(ix) ..... is the minimum value of $y = x^2$.

(x) Derivative of a constant is .....

(xi) If ..... of $y = x^3$ is $3x^2$, then ..... of $3x^2$ is x^3.

(xii) $\int_{-a}^{a} f(x)dx = 0$, if f(x) is ..... function.

(xiii) Area of the curve bounded by $y = f(x)$, x-axis and $x = a$ and $x = b$ is .....

(xiv) $\int_{-3}^{3} x^3 dx$ = ...

(xv) $\int 3x^2 \sin x^3 dx$ = ...

3. Choose the correct answer : 5

(a) Distance between (1, 2) and (4, 2) is

(i) $\sqrt{3}$ (ii) 2.1

(iii) 3 (iv) 4

(b) Gradient of the line passing through (6, 0) and (4, 2) is

(i) .1 (ii) – 1

(iii) 2 (iv) 1

(c) x-intercept of the straight line $5x + y = 10$ is

(i) 10 (ii) 5

(iii) 2 (iv) 1

(d) Radius of the circle $x^2 + y^2 - 2x - 4y - 4 = 0$ is

(i) 5 (ii) 1

(iii) 3 (iv) 2

(e) Focus of the parabola $y^2 = 4x$ is

(i) (1, 1) (ii) (0, 1)

(iii) (– 1, 0) (iv) (1, 0)

PART–B

4. Find limit (any *two*) : $3 \times 2 = 6$

(i) $\lim\limits_{x \to \infty} \dfrac{3x^2 + 2x - 1}{5x^2 - 2}$

(ii) $\lim\limits_{x \to 1} \dfrac{x^2 - 1}{x - 1}$

(iii) $\underset{x \to 1}{\text{Lt}}\, f(x)$ if $f(x) = 2x - 1, \quad x < 1$

$= x^2, \quad 1 < x$

(iv) $\underset{x \to 0}{\text{Lt}} \dfrac{\sqrt{1+x^2}-1}{x}$

5. Find $\dfrac{dy}{dx}$ (any *three*) : $2 \times 3 = 6$

(i) $y = \log (\cos^2 2x)$

(ii) $y = at^2, \quad x = 2at$

(iii) $x^y = y^x$

(iv) $x^3 + y^3 = 3xy$

(v) $y = x^4$ at $(-1, 1)$

(vi) $y = \sin 6x$

6. Find second order derivative of $f(x) = 2x - e^x + 1$. 2

7. Find equation to the tangent and normal to the curve $x^2 + x + y = 3$ at $(1, 1)$. 3

Or

Divide 15 into two parts so that the square of one multiplied by cube of the other is maximum.

8. (a) Evaluate any *three* : $3\times3=9$

(i) $\int \frac{\sin(\log x)}{x}\,dx$

(ii) $\int \frac{dx}{e^x + e^{-x}}$

(iii) $\int \cos 4x \cos 2x\,dx$

(iv) $\int \frac{dx}{x^2+4x+13}$

(v) $\int \frac{xe^x}{(x+1)^2}\,dx$

(b) Evaluate any *one* : 3

(i) $\int_0^{\frac{\pi}{2}} (a\cos^2 x + b\sin^2 x)\,dx$

(ii) $\int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x}\,dx$

9. Find the sum :

$$\lim_{x\to\infty}\left\{\frac{1^2}{n^3+1^3}+\frac{2^2}{n^3+2^3}+\frac{3^2}{n^3+3^3}+.....+\frac{n^2}{n^3+n^3}\right\}$$

3

Or

Find by integration the area of the triangle bounded by the line $4y - 5x = 0$, the x-axis and $x = 4$.

10. Answer any *two* : $2\times2=4$

(i) Find the ratio in which the point $(-11, 16)$ divides the line segment joining the points $(-1, 2)$ and $(4, -5)$.

(ii) Find polar coordinates of the point whose Cartesian coordinate is $(1, \sqrt{3})$.

(iii) Find the distance between the parallel lines $2x + 3y = 8$ and $2x + 3y + 16 = 0$.

(iv) Find centre and radius of the circle $x^2 + y^2 + 2x + 4y - 20 = 0$.

11. Answer any *three* : $3\times3=9$

(i) Find equation to the line passing through $(2, 1)$ and parallel to $2x - y + 3 = 0$.

(ii) A circle has its centre on the line $2x - 3y = 4$ and passes through the points (4, 3) and (– 2, 5). Find its equation.

(iii) The equation of a straight line is $6x - 8y + 5 = 0$. Write its gradient form, intercept form and perpendicular form.

(iv) Find the equation to the circle on the line (–1, –3) and (2, 4) as diameter.

(v) Trace the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$.